

*Philosophia Mathematica* (III) 00 (2012), 1–28.  
doi:10.1093/phimat/nks019

## Towards an Institutional Account of the Objectivity, Necessity, and Atemporality of Mathematics<sup>†</sup>

JULIAN COLE\*

I contend that mathematical domains are freestanding institutional entities that, at least typically, are introduced to serve representational functions. In this paper, I outline an account of institutional reality and a supporting metaontological perspective that clarify the content of this thesis. I also argue that a philosophy of mathematics that has this thesis as its central tenet can account for the objectivity, necessity, and atemporality of mathematics.

Recently, Jill Dieterle [2010] has offered a number of insightful criticisms of my [2008; 2009] attempt to defend the proposal that mathematical domains<sup>1</sup> exist in virtue of collective agreement. The most significant worries that she raises concern this proposal's ability to account for the objectivity and necessity of mathematics, though she [2010, fn. 14] also questions its ability to accommodate mathematics' atemporality. These worries are warranted, yet answerable. Indeed, the goal of this paper is to outline my current institutional philosophy of mathematics and indicate how it can account for the objectivity, necessity, and atemporality of mathematics.

Formally, the main thesis of my current account is:

Mathematical domains are freestanding institutional entities<sup>2</sup> that, at least typically, are introduced to serve representational functions.

To make the content of this thesis clear, I must explain some of the notions employed in stating it. In §1, I shall do so by outlining my conception of institutional reality. I want to be clear from the outset, however, that, while

<sup>†</sup> I thank Jacobo Asse, Randy Diapert, Jill Dieterle, Sol Feferman, Mike Gifford, Leonard Jacuzzo, John Kearns, Barbara Olsafsky, and an anonymous referee for helpful comments on earlier drafts of this paper, and Otávio Bueno and Thomas Ryckman for recent conversations about some of the topics discussed in it.

\* Department of Philosophy and Humanities, Buffalo State College, Buffalo, N.Y. 14222, U.S.A. colejc@buffalostate.edu

<sup>1</sup> More accurately, it is mathematical structures that exist in virtue of collective agreement. Yet, to avoid discussing structuralism, I shall write about mathematical *domains*, by which I mean collections of entities with properties and relations among them.

<sup>2</sup> I use *entity* to pick out all items that may legitimately fall within the range of a first-order variable, which includes those items that are normally called objects.

it is a consequence of this thesis that all *pure*<sup>3</sup> mathematical facts (e.g.,  $7 + 5 = 12$ ) obtain in virtue of collective agreement, this thesis does not entail that *all* mathematical facts obtain in virtue of such agreements. For instance, the fact that when you combine a collection with cardinality 7 and a collection with cardinality 5 you typically<sup>4</sup> obtain a collection with cardinality 12 does not obtain in virtue of collective agreement.

Now, my current account's explanation of mathematics' objectivity flows from its main thesis. By contrast, its account of mathematics' necessity and atemporality invokes a particular metaontological perspective, which, in turn, is central to the coherence of my conception of institutional reality; in §2, I shall outline this perspective. Following this, in §3, I shall sketch my institutional philosophy of mathematics and its account of the objectivity, necessity, and atemporality of mathematics. Unfortunately, further articulation and defense of this philosophy of mathematics will have to await another paper.

## 1. Institutional Reality

Many facts obtain and facets of reality<sup>5</sup> exist in virtue of collective agreement. Consider, for instance, that John Searle is the Slusser Professor of Philosophy at UC Berkeley, that John Burgess and Stewart Shapiro are friends, that certain of the pieces of paper in my wallet are \$20 bills, the border between France and Germany, Safeway Inc., the property of being a football team's quarterback, and the relation of being students in the same college class. In §1.3, I shall show that institutions are responsible for the existence of these facets of reality. Until then, I use 'institutional' as a synonym for 'exists in virtue of collective agreement'.

That there are institutional facets of reality raises a number of questions: Why do we undertake the agreements responsible for their existence? How can agreements be responsible for the existence of facets of reality? How much of reality is institutional? And, how is institutional reality sustained in existence? In this section, I shall answer these questions. Let me begin with the first.

### 1.1. *The Imposition of Function onto Reality*

We undertake the agreements responsible for the existence of institutional facets of reality as part of our assignment or imposition<sup>6</sup> of *function* onto

<sup>3</sup> *Pure mathematical facts* are those that wholly concern mathematical domains.

<sup>4</sup> Of course, when the entities in the two collections interact, they might combine or divide.

<sup>5</sup> Henceforth, *facet of reality* shall refer to facts, as well as entities, properties, relations, etc.

<sup>6</sup> Henceforth, I shall write simply of imposing function onto reality.

reality. It is common for such agreements to involve the imposition of a status onto some facet of reality (e.g., the aforementioned examples involve the imposition of the statuses ‘professor’, ‘university’, ‘friend’, ‘\$20 bill’, ‘country’, ‘border’, ‘corporation’, ‘football team’, ‘quarterback’, ‘student’, and ‘college class’). Facets of reality that have these types of statuses perform certain functions (e.g., roughly, the function of a university is to promote research and learning, and of a friend to render certain types of companionship and assistance). Indeed, we impose such statuses onto particular facets of reality to make them able to perform the associated functions.

All but one of the statuses listed in the previous paragraph carries *deontic powers* (henceforth, DPs) — a deontology,<sup>7</sup> i.e., rights, responsibilities, duties, obligations, requirements, entitlements, permissions, authorizations, etc., (e.g., ‘professor’ carries the *right* to assign grades to students and the *obligation* to do so fairly, while ‘college student’ carries the *responsibility* to complete assignments and *authorizes* one to attend certain college classes). The reason that such statuses carry DPs is that doing so facilitates the facets of reality on which they are imposed in performing their respective functions. Indeed, such facets of reality are only able to perform their respective functions because they carry DPs (e.g., Barack Obama is only able to perform the functions of the U.S. President because he has such DPs as the right to veto legislation passed by the U.S. Congress and the obligation to give the State of the Union address, while particular pieces of paper are only able to perform their financial functions because they carry the DPs associated with the status ‘\$20 bill’).

Label functions that facets of reality serve in virtue of having a status that carries DPs *status functions* (henceforth, SFs) and the associated statuses *SF statuses*. SF statuses actually carry *differential* DPs, i.e., DPs that accrue to some people and not others<sup>8</sup> (e.g., the DPs carried by ‘quarterback’ accrue only to quarterbacks, while those carried by ‘\$20 bill’ accrue only to that particular bill’s owner).<sup>9</sup>

Our primary interest is not in SFs; it is in the type of function performed by entities like borders. Borders are the borders *of* such items as parcels of land, nations, municipalities, etc. Frequently, the latter carry DPs (e.g., parcels of land carry the DPs of those who own them). Borders, rather than carrying this type of deontology, mark the locations of transitions in it (e.g., the border between my backyard and my neighbor’s marks the

<sup>7</sup> The exception is ‘border’; I shall explain why below.

<sup>8</sup> Since there is a ‘complementary’ relationship between various DPs, (e.g., rights imply obligations), the differential DPs that accrue to particular individuals when a SF is imposed provide others with ‘complementary’ DPs.

<sup>9</sup> The fact that everyone *can be* the owner of a given \$20 bill does not mean that the DPs that it carries do not accrue differentially to its actual owner.

location where the DPs that accompany her ownership end and those of my landlord begin). Such transitions are important for a variety of reasons. Consequently, we undertake an agreement that an entity — a border — exists whenever and wherever such a transition takes place. The function of such borders is to aid us in representing, analyzing, reasoning about, discovering truths concerning, *etc.* deontic transitions and related matters. Borders perform these functions by allowing us to represent these transitions by way of an entity with various features. And, as anyone who has compared the difficulty of assessing a nontrivial modal inference with and without using possible worlds to represent its premise(s) and conclusion knows, being able to represent a state of affairs *indirectly* by way of an entity with features when that state of affairs cannot be *directly* so represented can be tremendously helpful when it comes to analyzing, reasoning about, discovering truths concerning, *etc.* the state of affairs in question.<sup>10</sup>

Borders are not the only institutional entities that serve these kinds of functions. Let me provide three further examples. Corporations and universities are types of *organizations*, as are governments, leagues, clubs, associations, unions, and hospitals. Typically, organizations are associated with various individuals and groups of individuals, each of whom/which performs SFs that are relevant to the organization's operation (*e.g.*, corporations have CEOs, CFOs, Boards of Directors, shareholders, *etc.*, while governments have Presidents, Ministers, Parliaments, Citizens, *etc.*). Frequently, the exact configuration of an organization, *i.e.*, the exact relationship between the particular individuals and groups associated with it and their respective relationships to the organization itself, is complicated. This can make it difficult, for instance, to explain an organization's operation to people who are unfamiliar with it or to assess whether an organization would operate more efficiently if it were configured differently.

To facilitate these types of activities, we avail ourselves of an agreement to represent complex organizations as containing various *positions*, where each position corresponds to a locus of SFs relevant to the organization's operation (*e.g.*, we represent the U.S. Government as having three branches, which, in turn, contain such positions as Congress, the President, and the Supreme Court). Such positions can be divided into two sorts: *single-occupant positions* (*e.g.*, the positions of President and Speaker of the House), and *multiple-occupant positions* (*e.g.*, Congress and the Supreme Court). All such positions are institutional entities. Yet *single-occupant* positions, unlike *multiple-occupant* positions, do not, at least inherently, serve SFs, for it is the *occupants* of *single-occupant* positions who serve SFs (*e.g.*, it is the U.S. President, not the position of President, who has such DPs as the right to veto legislation passed

<sup>10</sup> A similar point can be made using the cardinalities of finite collections and natural numbers — see [Field, 1980].

by Congress, while it is the CFO of a corporation, not the position of CFO, who has such DPs as the responsibility to oversee that corporation's finances). In fact, inherently, single-occupant positions in organizations do nothing but facilitate our abilities to represent, analyze, reason about, discover truths concerning, *etc.* the configuration and operation of their respective organizations. Indeed, it is to allow them to serve these types of functions, and typically these types of functions alone, that we avail ourselves of the agreement that they exist.

The Laws of Chess govern particular *token-individuated games of chess*, *i.e.*, particular spatio-temporally instantiated activities in which two players make a sequence of moves in an attempt to achieve the goal of checkmate. Those who are proficient chess players know, however, that many features of such games are irrelevant to understanding them (*e.g.*, the speed and gusto with which the players make the sequence of moves in question); all that really matters is the sequence itself. Consequently, when we train people to become better chess players, or seek to establish such truths as 'it is impossible to force a checkmate with a king and two knights against a lone king', we tend to represent games of chess as mere sequences of moves. That is, we type-individuate them. *Type-individuated games of chess* are institutional entities whose inherent function is to facilitate our abilities to represent, analyze, reason about, discover truth concerning, *etc.* the game/institution of chess and particular tokens thereof.

No doubt, when Bram Stoker wrote about Count Dracula he was engaged in a pretense, a pretense he invited us to share by publishing *Dracula*. Yet when I claim 'there is a fictional character named Count Dracula who was written about by Bram Stoker', I am not engaged in a pretense nor do I speak non-literally. Rather, I (literally) assert something that is true. But what kind of entity is the fictional character Count Dracula? The most plausible answer of which I am aware is provided by [Amie Thomasson \[1999\]](#). She recognizes that there are a variety of purposes other than pure entertainment for which we use particular works of fiction, such as deepening our understanding of the people around us. Thomasson also recognizes that our ability to do so is significantly enhanced by our ability to talk and think about the characters introduced in these works independently of the works in which they were introduced. Further, she suggests that, to facilitate these purposes, we collectively agree that a collection of 'abstract artifacts', *i.e.*, abstract fictional characters, exists, where each such entity corresponds to a character introduced in some work of fiction. Indeed, we avail ourselves of this agreement to allow these 'abstract artifacts' to serve the same types of functions performed by boundaries, single occupant positions in organizations, and type-individuated games of chess.

So, there are a variety of institutional entities whose inherent function is to facilitate our abilities to represent, analyze, reason about, discover

truths concerning, *etc.* facets of reality that are not the entities in question. Label these types of functions, and all those in their vicinity, *representational* (henceforth, RF). Moreover, recall that the primary reason why we introduce facets of reality to serve RFs is to allow us to represent the world using intentional states that structure it into entities with features, for, as a result of the cognitive constitution of human beings at this evolutionary stage, we find it much easier to engage in the aforementioned types of activities using such states. Further, since the entities in question must have features to allow us to engage in these activities, their properties and relations are also institutional facets of reality that serve RFs.

In summary, then, human beings introduce institutional facets of reality as part of our imposition of function onto reality, and there are at least two classes of functions that we so impose: SFs and RFs.

## 1.2. *Declarations*

For the purpose of addressing the issue of how a collective agreement might be responsible for the existence of a facet of reality, recognize that institutional facets of reality are the products of representational acts<sup>11</sup> that have the same logical form as Declarations, where *Declarations* are speech acts that represent reality as including some facet and, in so doing, bring that facet into existence. For example, when one marries two people by uttering ‘I now pronounce you man and wife’ in an appropriate context or resigns from a position by uttering ‘I resign’ in an appropriate context. As [John Searle \[2010, p. 12\]](#) explains,

[Declarations] are cases where we change reality to match the . . . content of the speech act . . . . But . . . we succeed in so doing because we represent . . . reality as being so changed.

Sometimes we impose SFs and RFs using actual Declarations. The captain of a soccer team might, for instance, Declare ‘Tom, you will be our goalkeeper’. Yet this is not the only way in which Tom can become a team’s goalkeeper; he might simply stand by the relevant goal and, thereby, cause all relevant parties to start talking and thinking about him as the goalkeeper. Similarly, occasionally people formally Declare that someone is their friend, yet typically they just start talking and thinking about that person differently. As [Searle \[2010, p. 13\]](#) puts it, on many occasions, ‘we just linguistically treat or describe, or refer to, or talk about, or even think about an object in a way that creates a reality by representing that reality as

<sup>11</sup> [[Searle, 2010](#), Chapter 2] explicates the formal meaning of ‘representational’ in this claim.

created'. Henceforth, 'Declaration' shall refer to *all* such representational acts rather than merely Declarations in the aforementioned narrow sense.

Such straightforward Declarations are not the only means by which facets of reality take on SFs and RFs, however. They also do so by way of *standing Declarations*, *i.e.*, Declarations that remain in force for an extended period of time and, during this period, specify conditions the fulfillment of which will suffice for the existence of a certain facet of reality. For instance, all four of the classes of entities that we discussed in §1.1 that serve RFs are the products of standing Declarations. The contents of those Declarations are, roughly: whenever and wherever a piece of land is parceled off to serve some function a further entity exists — that parcel's border; whenever a single individual serves SFs relevant to a complex organization's operation a corresponding entity exists — a single-occupant position in that organization; whenever the Laws of Chess permit a token-individuated game of chess in which the players make a particular sequence of moves a corresponding entity exists — a type-individuated game of chess; and whenever an author introduces a character in a work of fiction a corresponding entity exists — an abstract fictional character. Of course, these are merely the standing Declarations responsible for the existence of the entities in question; related standing Declarations specify their respective features and, in consequence, facilitate their being able to perform their respective RFs.

Each of the systems of standing Declarations just described is accepted even though there is no established formal codification of them. Similar uncoded systems of standing Declarations govern the imposition of certain simple SFs (*e.g.*, in numerous cultures, there are uncoded standing Declarations that automatically pass leadership from fathers to eldest sons). For more complicated SFs, however, it is common for the relevant systems of standing Declarations to be codified (*e.g.*, many games have codified rules, while corporations have various corporation codes). The State of California's Corporation Codes contain the standing Declarations relevant to the status 'Californian corporation'; Section 200(a) of those Codes reads:

One or more natural persons, partnerships, associations or corporations, domestic or foreign, may form a corporation under this division by executing and filing articles of incorporation.

These Codes also specify the DPs carried by various classes of Californian corporations. Indeed, in general, the DPs carried by a particular SF status are specified in the system(s) of standing Declarations responsible for the imposition of that status.

In summary, then, institutional facets of reality come to exist in virtue of our collectively adopting systems of (standing) Declarations, where such systems govern particular types of activities. California's Corporation Codes, for instance, govern corporate activities in the State of California, while the FIDE Laws of Chess govern chess-playing activities in the context of official FIDE matches. Similarly, the four systems of standing Declarations highlighted above govern the activities of representing, analyzing, reasoning about, discovering truths concerning, *etc.*, respectively, transitions in deontology, the configuration and operation of complex organizations, the game/institution of chess and particular tokens thereof, and the relevance of characters introduced in works of fiction to our understanding of the world.

In fact, the (standing) Declarations in the aforementioned systems are special types of rules. What makes them special is that they not only regulate the relevant activities, they also *make possible the existence of certain facets of reality* (e.g., California's Corporation Codes make possible the existence of Californian corporations, while the Laws of Chess make possible checkmating an opponent). Searle labels rules of this type *constitutive* and contrasts them with *regulative rules*, *i.e.*, rules that merely regulate activities (e.g., 'raise your hand if you want to talk').<sup>12</sup>

### 1.3. *Institutions and Institutional Facets of Reality*

Formally, an *institution* is any collection of activities governed at least in part by a system of constitutive rules, while a facet of reality is *institutional* if and only if its existence is made possible by the constitutive rules of an institution. Moreover, institutional facets of reality should be contrasted with *brute facets of reality*, *i.e.*, facets of reality that exist independently of all institutions (e.g., electrons and the fact that the Earth is roughly spherical). We have been discussing two types of institutions: institutions that impose SFs onto reality and institutions that impose RFs onto reality. Label these, respectively, *SF institutions* and *RF institutions*. Additionally, note that institutional entities fall into two categories: those that are *freestanding*, *i.e.*, neither 'identical to' nor 'constituted by' any entity or collection of entities that exists independently of the institution responsible for the existence of the institutional entity in question (e.g., type-individuated games of chess and abstract fictional characters), and those that are *non-freestanding*, *i.e.*, 'identical to' or 'constituted by' some entity or collection of entities that is identifiable independently of the institution responsible for the existence of the institutional entity in question (e.g., \$20 bills and the 44<sup>th</sup> President of the U.S.A.).

<sup>12</sup> Officially, regulative rules are standing Directives rather than standing Declarations.

SF institutions are ubiquitous. Searle [2010, p. 91], for instance, lists the following categories of nonlinguistic SF institutions while outlining the scope of institutional reality:

Governmental institutions: legislature, executive, judiciary, military, police.

Sports institutions: the National Football League, amateur baseball teams, local sports clubs.

Special-purpose institutions: hospitals, schools, universities, trade unions, restaurants, theaters, churches.

Economic institutions: industrial corporations, brokerage houses, real estate agencies, business partnerships.

General-purpose structural institutions: money, private property, marriage, government.

Unstructured informal (mostly) uncodified institutions: friendship, family, love affairs, parties.

I contend that RF institutions are almost as ubiquitous as SF institutions. Above, to minimize controversy, I concentrated on RF institutions that surround constructed entities. Yet, there is no reason to limit them in this way. The aim of this paper, for instance, is to argue that mathematics is a RF institution or, more accurately, a collection of such institutions. I believe that similar RF institutions are responsible for the existence of possible worlds, which serve RFs with respect to possibilities, propositions and meanings, which serve RFs with respect to the contents of thoughts and assertions, and universals, which serve RFs with respect to similarities in the world.

#### 1.4. *Collective Recognition*

Roughly, institutional facets of reality are sustained in existence by a relevant group of people collectively recognizing or accepting<sup>13</sup> their existence. Moreover, for facets of reality that serve SFs, this group collectively recognizes their existence by way of collectively recognizing the DPs carried by their SF statuses (*e.g.*, roughly, Tom is a team's goalkeeper if and only if he is collectively recognized to have the relevant DPs, while I am a professor if and only if I am collectively recognized to have the DPs carried by this status). In fact, something similar is true of institutional facets of reality that serve RFs. In order to appreciate this fully, however, we need to explore DPs and their relationship to RFs further.

<sup>13</sup> Henceforth, I shall simply write collectively recognizing.

In English there is a collection of mechanisms for committing oneself to the existence of particular facets of reality,<sup>14</sup> though there is some controversy over what, precisely, they are. Plausible examples include ‘X exists’ and ‘there are Xs’. Whatever these mechanisms are, an English speaker is only *authorized* to use one of them when the facet of reality in question exists and will likely be *required* to use one of them to commit himself or herself to its existence. Similarly, in appropriate contexts, anyone who is engaged in the activity of formalizing natural-language assertions using first-order logic is *obligated* to include an existent entity within the range of his or her first-order variables. Label DPs of these types *existential* since one way to recognize that *any* facet of reality exists is to recognize that it carries these DPs. Indeed, these DPs are carried by *all* facets of reality. For instance, when an entity is introduced to serve certain SFs (*e.g.*, when a LLC is introduced to limit the liability of various people), it carries not only the differential DPs associated with its SFs, but also existential DPs. Further, existential DPs accrue to *everyone*, which distinguishes them from the differential DPs carried by SF statuses (*e.g.*, *anyone* has the right to claim of any existent entity that it exists).

For our purposes, the key observation in the previous paragraph is that the existence of institutional facets of reality whose inherent function is to serve RFs can be (and is) collectively recognized by people collectively recognizing the existential DPs that they carry. Moreover, all that is required in order for such facets of reality to be able to serve their RFs is that we *permit* people to talk and think about the world by way of them. In other words, all that is required in order for facets of reality to serve RFs is for those who wish to use them so to have the *right* to commit themselves ontologically to the facets of reality in question. For instance, provided that we permit people to talk and think about the institution of chess and particular token-individuated games of chess by way of type-individuated such games, the latter entities can serve their RFs, while provided that we allow people to talk and think about the configurations of complex organizations by way of positions in them, single-occupant such positions can serve their RFs.

### 1.5. *The Temporal and Modal Indexicality of Institutional Facets of Reality*

Section 200(c) of California’s Corporation Codes reads, ‘The corporate existence begins upon the filing of the articles and continues perpetually, unless otherwise expressly provided by law or in the articles’. Clearly,

<sup>14</sup> I have in mind facets of reality that already exist, not those that are being Declared to exist by means of the activity in question.

those who wrote this Code recognized that, in general, we have no idea how long a given Californian corporation might be in business. So they chose to stipulate that, in general, Californian corporations will continue to exist *perpetually* to ensure that their existence would never be shorter than needed to serve their functions. This is an example of a relatively harmless liberty with time *for our pragmatic purposes*. I want to demonstrate that some significantly more controversial such liberties are equally harmless.

First, recognize that the RF institutions discussed above are responsible for the existence of facets of reality that existed *prior to* those institutions themselves. To see this, observe that these institutions all introduce facets of reality to aid us in representing states of affairs that are not the said facets. Yet, almost certainly, in each case, there was a lag between the obtaining of the types of states of affairs that these facets of reality serve to represent — transitions in deontology, complex organizations, token-individuated games of chess, and characters in works of fiction — and people's thinking to represent those states of affairs by way of newly introduced facets of reality (*e.g.*, almost certainly, (early versions of) the Laws of Chess determined the permissibility of various sequences of chess moves before anybody thought to type-individuate games of chess). Further, almost certainly, just after introducing these new facets of reality, the relevant people did not restrict their uses of them to merely representing current and future states of affairs, they also used them to represent states of affairs in their pasts. For instance, when people first used type-individuated games of chess to represent token-individuated games, they almost certainly used them to represent not only current and future such games but also ones played in their pasts, something which they could have done only if *type-individuated* games of chess existed in their pasts.

Now, it might seem that it is impossible for an institution to be responsible for the existence of a facet of reality that existed prior to it. Yet it is not. Since Declarations are responsible for the existence of institutional facets of reality and people can represent the past as being many ways, they can, should it serve their purposes, *Declare* and *collectively recognize* the past to be one of those ways. This type of retroactive application for representational purposes is familiar to us all; it is a feature of many linguistic institutions (*e.g.*, we can all use English to talk about prehistoric times or the beginning of the universe). A difference between linguistic and RF institutions, however, is that institutional facets of reality that serve RFs need to exist at time *t* in order to serve their RFs with respect to *t*. Thus, if a group wishes to use a facet of reality collectively to represent, analyze, reason about, discover truths concerning, *etc.* a time prior to the existence of the RF institution responsible for its existence, they need collectively to recognize that that institution makes it the case that that facet of reality existed at that time.

While rare, temporal liberties that involve retroactive application for pragmatic purposes also occur for facets of reality that serve SFs. This is illustrated by John Searle's [2010, Chapter 8] discussion of human rights. For instance, while it is *now* an institutional fact that slaves in Ancient Greece had human rights that their owners failed to recognize, there was no such fact *at that time*. This fact obtains *at this time* since there is *now* a widely accepted SF institution that has a constitutive rule that specifies that meeting certain biological conditions, which the slaves in Ancient Greece clearly met, (was, is, and will be) sufficient for an individual's having human rights. This fact did not obtain *at that time* since *then* there was no such institution. It is worth noting, however, that two hundred years ago there was such an institution. Moreover, the collection of human rights that this institution's participants *then* took the slaves in Ancient Greece to have possessed is different from the collection that its *current* participants take them to have possessed. So, at different times, an institution can constitute a facet of reality differently. A second illustration of this point is provided by the practice in Major League Baseball of teams' placing players on the official disabled lists retroactively. Suppose that a player gets injured and that his team's medical staff cannot assess whether he will recover relatively quickly. Since the team can only place the player on either a 15 day or 60 day disabled list, it may wish to wait before it takes either action to see how his recovery proceeds. According to MLB regulations, it can place the player on either list retroactively any time within the 10 days after the injury. Thus, on the Friday after his injury a player might not be officially on the 15 day disabled list, yet on the following Monday he might be retroactively placed on this list going all the way back to the previous Monday. Hence, who appears on the official disabled lists for a given day can vary over time. So, in general, roughly, *at time*  $t_1$ , an institution is responsible for the existence of a facet of reality with certain features at time  $t_2$  or atemporally if and only if, at  $t_1$ , its participants collectively recognize the DPs relevant to the existence of that facet of reality with those features at  $t_2$  or atemporally.

Label the collection of times at which some facet of reality exists or its atemporality that facet's *temporal profile* and the collection of possible worlds in which it exists or its amodality its *modal profile*. The above shows that, for institutional facets of reality, we need to distinguish the *constituting institution's* temporal profile — the temporal profile of the activities responsible for the existence of the facet of reality in question — from the *constituted facet's* (potentially varying) temporal profile. Observe, moreover, that we can also *Declare* and *collectively recognize* that institutional facets of reality have whatever modal profile best serves our purposes. Thus, roughly, in the actual world, at time  $t$ , an institution is responsible for the existence of a facet of reality with certain features with a given temporal and modal profile if and only if, in this world, at  $t$ ,

its participants collectively recognize the DPs relevant to the existence of that facet of reality with those features with that temporal and modal profile. Finally, note that earlier in this section I assumed that in *this world*, at *this time*, we collectively recognize that various institutional facets of reality with specific features exist with particular temporal and modal profiles.

## 1.6. Summary

In summary, then, institutions are collections of activities governed by constitutive rules. Institutions can be responsible for the existence of institutional facets of reality, because such rules are standing Declarations. At least two types of standing Declarations serve as the constitutive rules of institutions: those that make possible the imposition of RFs onto reality and those that make possible the imposition of SFs onto reality. Both RFs and SFs provide particular individuals with specific DPs. And, roughly, institutional facets of reality are sustained in existence by collective recognition of their existence by way of collective recognition of the DPs that they carry. Further, for convenience, let me repeat the two acronyms that I introduced above that will be important later in this paper: DP — deontic power, and RF — representational function.

## 2. Reference and Metaontology

### 2.1. A Collocation Puzzle

Consider the following objection to the above account of institutional reality:

... if you suppose that there are two classes of objects, [institutional] and non-[institutional], you immediately get contradictions of the following sort: In my hand I hold an object. This one and the same object is both a piece of paper and a dollar bill. As a piece of paper it is a non-[institutional] object, as a dollar bill it is a[n] [institutional] object. So which is it? The answer, of course, is that it is both. But to say that is to say that we do not have a separate class of objects that we can identify with the notion of [institutional] object. Rather, what we have to say is that something is a[n] [institutional] object only under certain descriptions and not others ... [Searle and Smith, 2003, p. 302]

At the heart of this objection<sup>15</sup> is a collocation puzzle that is an instance of a more general puzzle. For instance, since antiquity, philosophers have

<sup>15</sup> What Searle is doing here is challenging the idea that there are genuinely *institutional* entities; he is suggesting, rather, that reality contains only brute entities that can sometimes

worried about the relationship between statues and collocated lumps of clay, while, recently, issues of collocation have been used to question the existence of *all* of the medium-sized spatio-temporal entities in our folk ontology. Fortunately, as Amie Thomasson [2007] shows, we can solve the general collocation puzzle, and thus undermine the aforementioned objection to my account of institutional reality, without relinquishing our folk ontology, including institutional entities.<sup>16</sup> The key elements of her solution are a hybrid theory of reference [Chapter 2] and a related metaontological perspective [Chapter 3]. Since this metaontological perspective, or a similar one, underwrites my institutional philosophy of mathematics, I shall outline it and the hybrid theory of reference on which it relies before I apply the framework developed in §1 to mathematics.

## 2.2. A Hybrid Theory of Reference<sup>17</sup>

The central insight concerning reference that the aforementioned hybrid theory embeds is the following: for a term to succeed in referring to a statue rather than a lump of clay or a piece of paper rather than a dollar bill, while its reference was being grounded, the person or people who so grounded it must have had in mind a category of entity to which he/she/they intended to refer, for otherwise, its reference would be radically indeterminate. Moreover, something akin to this is true for all nominative terms. As Thomasson puts it, ‘reference is only unambiguously established to the extent that our nominative terms are associated with high-level conceptual content establishing what category of entity is to be referred to by the term, if it refers at all’ [2007, p. 38]. Further, by establishing this relationship to a particular category of entity or, more precisely, *categorical concept*, a term’s ‘high-level conceptual content’ achieves two things, at least where both are appropriate.<sup>18</sup> First, it establishes the term’s (*frame-level*)<sup>19</sup> *application conditions*; that is ‘certain very basic conditions under which the attempted grounding would or would not be successful in establishing reference’ [p. 39]. Second, it establishes the term’s (*frame-level*) *coapplication conditions*; that is ‘(supposing the term to have been successfully applied) [it establishes] under what conditions the term would be applied again to one and the same entity’ [p. 40]. Informally, these

be appropriately described as institutional. Yet this is incompatible with recognizing, as my view must, that there are *freestanding* institutional entities.

<sup>16</sup> I favor Thomasson’s solution to this puzzle since, among other things, unlike its competitors, it accommodates freestanding institutional entities — see [Thomasson, 2003].

<sup>17</sup> Thomasson largely borrows this theory of reference from [Devitt and Sterelny, 1999].

<sup>18</sup> Henceforth, I shall leave this qualification implicit.

<sup>19</sup> ‘Frame-level’ is used to emphasize that these conditions are *conceptually* relevant to how the term refers.

frame-level conditions constitute the *rules of use* of the term in question. Furthermore, establishing the aforementioned relationship has the aforementioned effects, because categorical concepts have *stipulated/legislated* application and coapplication conditions that reflect our *pragmatic interests* in the category of entity in question.

Now, it is important that we carefully distinguish the application and coapplication conditions of terms/concepts and recognize their relative independence from one another, for certain of our terms/concepts have application conditions yet lack coapplication conditions (e.g., ‘smooth’) and the application conditions of two terms/concepts can coincide while their coapplication conditions differ (e.g., ‘Hamlet’ and ‘token of Hamlet’).

Moreover, the theory of reference under discussion combines the above insight into how the reference of a nominative term is grounded with a straightforwardly causal account of how it is *borrowed* by users who were not involved in said grounding, which allows it to accommodate the type of speaker ignorance and error that provided the original motivation for causal theories of reference. Furthermore, this theory does not require that even competent users of a nominative term/categorical concept be able to state its application and coapplication conditions, merely that they have a *tacit* understanding of these conditions (e.g., they should be able to use the term/concept correctly, and be able to correct others when they do not).

Additionally, the aforementioned categorical concepts are grouped in hierarchies of varying specificity, where fulfillment of the application and coapplication conditions of a more specific concept is sufficient for the fulfillment of these conditions of a less specific concept. And, the hierarchical nature of these categorical concepts ensures that there are *analytic interrelations* between them and their associated terms. For instance, since fulfillment of the application conditions of ‘\$20 bill’ is sufficient for the fulfillment of the application conditions of ‘piece of paper’, ‘All \$20 bills are pieces of paper’ is an analytic truth.

In light of W.V.O. Quine’s well-known attack on the analytic-synthetic distinction, the latter feature of this theory might trouble some. Yet the type of analyticity that it invokes is not one that Quineans should find problematic. Primarily, this is because analytic statements of the type countenanced by this theory do not perform the epistemic functions that Carnap and his predecessors ascribed to them. In particular, the aforementioned stipulations/legislations are *revisable* in light of at least two factors: empirical discoveries and modifications of our pragmatic interests in the category of entity in question. As Thomasson [2007, p. 37] remarks:

Revisions in the case of analytic statements must merely be considered to be revisions based on proposed reconfigurations of our conceptual system rather than revisions based on direct conflict with

experience. We do not *discover* that they are false . . . but we may *decide* to alter them, in order to make the system as a whole work better.

Moreover, in most cases, the factors that lead to such ‘reconfigurations of our conceptual system’ should be assessed holistically rather than individually.<sup>20</sup>

Next, let me show how this theory dissolves the collocation puzzle. Owing to how reference works according to it, it is only possible to count entities once one has specified a category (or collection of categories) of entity to be counted. Furthermore, when the concepts associated with particular categories of entity are analytically interrelated, we rarely count them at the same time (*e.g.*, we rarely count both a statue and the lump of clay that ‘constitutes’ it or both a dollar bill and the piece of paper that ‘constitutes’ it). Thus, the only reason why Searle, in his exchange with Smith, is inclined to believe that there is a single entity in his hand — something which is *both* a piece of paper and a dollar bill — rather than two entities in his hand — a piece of paper *and* a dollar bill — is that the relevant concepts are analytically interrelated. Yet, according to the theory of reference under discussion, Searle does have two entities in his hand — a piece of paper *and* a dollar bill. Moreover, the same observations can be made about any pair of collocated entities. Thus, the theory of reference under discussion dissolves the collocation puzzle by showing that it presupposes something that is not true, *i.e.*, that collocated entities are identical.

### 2.3. *Metaontology*

The theory of reference outlined in §2.2 has important implications for our understanding of various metaphysical claims, for the rules of use of nominative terms establish the truth-conditions of such claims concerning the entities (sorts of entity) to which those terms refer. In particular, the application conditions of a singular nominative term fix what are commonly labeled the existence conditions of the entity, *if any*, to which that term refers, while the application conditions of a sortal term fix the conditions under which it is true, *if it is true*,<sup>21</sup> that there is something of that sort. Likewise, the coapplication conditions of nominative terms fix the truth-conditions of identity claims made using those terms and, hence, fix what are commonly labeled the identity conditions of the entities (sorts) to which such terms refer. Moreover, since the existence and identity conditions of a given entity (sort) jointly determine what are commonly labeled

<sup>20</sup> Those readers interested in a more detailed defense of this theory against Quinean worries concerning the analytic-synthetic distinction should consult [Thomasson, 2007].

<sup>21</sup> Henceforth, I shall leave qualifications like this implicit.

its (that sort's) persistence conditions, in fixing the rules of use of the terms that refer to an entity (sort), we also fix its (that sort's) persistence conditions. Furthermore, the rules of use of a nominative term also fix the modal features of the entities (sorts) to which that term refers (*e.g.*, bachelors are necessarily men since fulfillment of the application conditions of 'bachelor' is sufficient for fulfillment of the application conditions of 'man').

So, the hybrid theory of reference developed in §2.2 maintains that the existence, identity, and persistence conditions of the entities (sorts) to which we refer, as well as their modal features, are fixed by the semantic rules for how we use our nominative terms, which, in turn, are fixed by our pragmatic stipulations/legislations concerning the application and coapplication of the relevant categorical concepts. In fact, claims about the existence, identity, persistence, and modal features of entities (sorts) express, in the object-language, truths about the conditions under which our concepts/terms apply and reapply that would, otherwise, have to be stated in a meta-language. Indeed, according to the perspective I am articulating, it is the primary function of such claims to convey this information. Note, however, that such object-language metaphysical claims *use* the relevant terms rather than *mention* them. Thus, such claims are *not* reports or descriptions of the application and coapplication conditions of our concepts/terms.

Additionally, observe that uses of alethic modal terms such as that of 'necessarily' in 'necessarily, bachelors are men' mimic a widespread practice of using deontic modal terms to convey rules. The canonical way of stating the constitutive rules of games, for instance, is as imperatives. Yet, for various reasons,<sup>22</sup> we sometimes state them as indicatives. Unfortunately, when we do so, we risk their being misconstrued as mere descriptions of how the game is to proceed rather than as rules. So, to avoid misconstrual, we incorporate deontic modal terms to convey their force (*e.g.*, the second rule of Scrabble can be expressed 'Each player *must* complete his or her turn by counting and announcing the score for that turn.'). Moreover, by using modal terms, we are able to state rules that convey permissions in addition to those that convey requirements (*e.g.*, 'A player *may* skip a turn.').<sup>23</sup> Label uses of alethic modal terms to communicate information about the application and coapplication of our concepts/terms *internal*. In §3, I shall contrast them with *external* uses of such terms, which relate to available (external) possibilities. In understanding the latter notion, it is helpful to build on the aforementioned analogy with games. Consider, for instance, a member of the rules committee of some game claiming that we *could* change one of its rules to be *Y* rather than *X*; he or

<sup>22</sup> For instance, it is more polite, makes the addressee clear, and facilitates basic reasoning from the rules.

<sup>23</sup> Further details about the modal component of the perspective I have articulated can be found in [Thomasson, 2010].

she would be highlighting an alternative (external) possibility for how the game might be played.

Now, the type of priority given to semantic rules/conceptual stipulations by the theory of reference and associated metaontological perspective under discussion ensures that basic claims about the existence, identity, persistence, and modal features of the entities (sorts) to which we refer are analytic and, hence, discoverable by a kind of conceptual analysis. Indeed, according to this perspective, label it *modal conceptualism*, all (internal) modal truths concerning the entities (sorts) to which we refer are ultimately based on analytic truths in the sense that they are either analytic themselves or the result of combining an analytic truth with empirical ones, where the empirical truths in question ‘fill in’ information relevant to the rules of use of the nominative terms used in stating the modal truth in question.<sup>24</sup> Theories similar to modal conceptualism, under the label modal conventionalism, have been developed and criticized in the literature.<sup>25</sup> Similar criticisms have been leveled against conventionalist accounts of analyticity that resemble the account of analyticity outlined above.<sup>26</sup> Thomasson [2007, Chapter 3] extensively discusses various objections to these types of conventionalism and shows that they apply neither to the above account of analyticity nor to modal conceptualism. To avoid getting sidetracked, I refer the skeptical reader to her work.

2.3.1. Applying this Metaontological Perspective to Institutional Reality  
In articulating the metaontological perspective outlined in §2.3, I primarily discussed its application to brute facets of reality, for I wanted to establish its tenability to those who will assess it in this context. Yet we are also able to make conceptual stipulations that serve to underwrite institutional facets of reality. In so doing, we must use our understanding of the *functions* that we are seeking to impose to generate new categorical concepts with which we can associate application and coapplication conditions that reflect those functions.<sup>27</sup> We can then make whatever Declarations are necessary to introduce facets of reality to serve the said functions.

Moreover, when we investigate the constitutive rules responsible for the existence of, for instance, particular freestanding institutional entities, we find that such rules are sensitive to the need to specify the relevant application and coapplication conditions, even if they are misleadingly expressed in the object-language. Consider, for example, Sections 200(a)

<sup>24</sup> Modal conceptualism’s acknowledgement of the second class of modal truths allows it to account for *a posteriori* necessities.

<sup>25</sup> Positively, see, e.g., [Sidelle, 1989]; critically, see, e.g., [Elder, 2004] and [Rea, 2002].

<sup>26</sup> See, e.g., [Boghossian, 1997] and [Sider, 2003].

<sup>27</sup> This requires the social construction of the category/kind in question.

and (c) of California's Corporation Codes, quoted in §1.2 and §1.5, respectively. These give, respectively, existence conditions for Californian corporations, and information relevant to the identity and persistence conditions of such corporations.

### 3. Towards an Institutional Account of Mathematics

#### 3.1. *An Overview*

We are finally in a position to understand my institutional philosophy of mathematics. To repeat, its main thesis is:

Mathematical domains are freestanding institutional entities that, at least typically, are introduced to serve RFs.

The idea that mathematical domains serve something like RFs is not new. It can be found in, at least, Hartry Field's [1980] defense of nominalism, and Stephen Yablo's [2005] and Mary Leng's [2010] fictionalism. Note, however, that according to my account, mathematical domains are *genuine existents*. Specifically, the *central constitutive rule of mathematics* — roughly, someone's undertaking an ontological commitment to a mathematical domain during the course of an investigation suffices for the existence of the domain in question — is responsible for their existence. Of course, such an individual is able to undertake such a commitment only if he or she has constructed an appropriate concept that, at least roughly, coherently characterizes the said domain. Furthermore, like all institutional facets of reality, such domains are sustained in existence, *if* they are sustained in existence, by ongoing, typically collective,<sup>28</sup> recognition of the DPs that they carry.

Now, it is useful to think of mathematical reality as having been constructed in layers. In the first layer are domains, such as the natural numbers and Euclidean planes, that were introduced to perform RFs with respect to non-mathematical facets of reality.<sup>29</sup> Yet, having introduced such domains, we became interested in them, and began to investigate them, independently of their RFs. Further, while so doing, we found it useful to introduce additional domains — a second layer — to perform RFs with respect to the domains in the first, though it might be better to describe

<sup>28</sup> My account allows for the possibility of an isolated individual's introducing a new mathematical domain provided that others would, were they to become aware of it, recognize the existential DPs that it carries.

<sup>29</sup> Included in these facets of reality are non-mathematical institutional facets of reality. Consider, for instance, the motivation provided by the insurance and gaming industries for the development of probability theory.

some of the domains in this second layer as *extensions* of those in the first layer rather than as elements of a new layer.<sup>30</sup> And, as before, we took an interest in, and began to investigate, these domains independently of their RFs, which led to our introducing a third layer/further extensions, *etc.*

Additionally, almost certainly, our introduction of mathematical domains has not come to an end, for, almost certainly, we will continue to introduce them to aid us with various investigations. Moreover, if they, or similar domains that better perform the functions for which the originals were introduced, do aid with such investigations, the existential DPs that they carry will be collectively recognized, which will ensure their ongoing existence.

Undoubtedly, all early mathematical domains were introduced to serve RFs. I qualify my main thesis, however, since I am unsure that the same is true of *all* mathematical domains. As mathematicians sometimes claim that they continue their investigations of particular domains more for their aesthetic virtues than their contributions to broader mathematical projects, the primary interest of some mathematical domains might not lie in their RFs. So perhaps some such domains were not introduced to perform RFs, or not *merely* to perform RFs. I suspect, though, that all mathematical domains *can* perform RFs, for this seems to be part of what it is for a domain to be *mathematical*. But should there be mathematical domains that cannot serve RFs, note that, in light of my observation in §1.1 concerning our preferences for conducting investigations using intentional states that represent reality as containing entities with features, any investigation of such a domain would, by itself, give us reason to undertake an ontological commitment to it. Hence, the central constitutive rule of mathematics would guarantee its existence.<sup>31</sup>

<sup>30</sup> The illustrations of RFs in §1 differ from those relevant to the second and subsequent layers of mathematical reality. For instance, mathematicians frequently introduce domains that are closed under various important operations (*e.g.*, the complex numbers). At least part of the function of such domains is to aid mathematicians in their investigations of the previous layer(s) of mathematical reality, though, in so doing, they may also help shed light on non-mathematical facets of reality. The important point, however, is that such domains do serve RFs.

<sup>31</sup> Coincidentally, a similar argument can be provided for mathematical domains that can serve RFs. In addition, the central constitutive rule of mathematics does not entail that mathematical investigations cannot be tentative; there are a variety of senses in which they can be. For example, those involved in such an investigation might be tentative about the coherence of their conception of a given domain, tentative about whether their current characterization of a given domain is optimal, tentative about whether the domain that they are investigating serves the functions that are of interest to them, tentative about whether the broader mathematical community will find their investigation interesting or relevant, *etc.* I thank an anonymous referee for suggesting that I comment on this matter.

### 3.2. A Simple Illustrative Example: Natural Numbers

To ensure that the reader understands my account of mathematics, let us consider its application to the natural numbers. It should be unsurprising that these entities were introduced to serve RFs with respect to finite collections. At first, the collections were relatively small and of spatio-temporal entities of particular categories. Moreover, most of the entities in these categories were brute rather than institutional. So, in introducing the natural numbers as a tool for representing the cardinality of various collections, our predecessors did *not* bring the entities in those collections into existence; these entities already existed. Further, over time our predecessors expanded the RFs of natural numbers so that the represented collections could be larger, contain entities from multiple categories, and include past, future, and non-spatio-temporal entities. Furthermore, given that no upper limit could be put on the size of the collections that were of interest to our predecessors, they introduced a full  $\omega$ -sequence of entities to serve the relevant RFs. Indeed, eventually, our predecessors recognized that they wished to use natural numbers to perform their RFs with respect to all (externally) *possible* finite collections, where, undoubtedly, to some extent, their understanding of what finite collections were possible coevolved with their understanding of the RFs of natural numbers.

Now, the expansion over time of the RFs of natural numbers required an evolution of the application and coapplication conditions of the categorical concepts/terms associated with such numbers. Specifically, these conditions evolved to make appropriate the use of these concepts/terms in contexts in which increasingly heterogeneous collections were conspicuous. A second, more fundamental, evolution of these conditions occurred when our predecessors decided to use these concepts/terms in contexts in which no collection was conspicuous; as I put it above, they became interested in, and began to investigate, natural numbers independently of their RFs. In reality, this transition amounted to their establishing a second set of uses for the relevant concepts/terms. For convenience, and inspired by the related distinction within mathematics, label the first set of uses of these concepts/terms *applied* (e.g., the use of ‘two’ in ‘there are two people in front of me’) and the second set *pure* (e.g., the uses of ‘two’ in ‘two plus two is equal to four’).

The aforementioned distinction is related to a second distinction, that between what I shall label the *immediate* and the *mediated* RFs of natural numbers. To illustrate this distinction, consider determining the cardinality of a large finite collection, entirely made up of non-overlapping smaller collections, by first determining the cardinality of the smaller collections and representing these by way of the appropriate natural numbers and then using arithmetic to determine the sum of these numbers. The link in this example between the cardinality of each collection and a particular natural

number illustrates an immediate RF of the natural numbers, while its use of arithmetic a mediated RF of these numbers.

### 3.3. *On the Atemporality and Amodality of Mathematics*

This simple example illustrates three important truths. First, typically, the RFs of mathematical domains are *universal*, *i.e.*, such domains typically (come to) serve their RFs with respect to *all (external) possibilities* of some type, where our understanding of what external possibilities of this type there are might coevolve with our understanding of the RFs of the relevant domain.<sup>32</sup> Second, mathematical concepts/terms typically have both pure and applied uses. Third, mathematical domains typically serve both immediate and mediated RFs.

As one might expect given my discussion of the influence of pragmatic purposes on the temporal and modal profile of institutional facets of reality — see §1.5 — these three truths have important implications for how mathematical domains are constituted. Specifically, if the domains in the first layer of mathematical reality are to serve their *mediated* RFs for all possibilities of a relevant type, past, present, future, and in any location, then the rules governing *pure* uses of the terms that refer to those domains and their components must have neither spatio-temporal nor modal restrictions placed on them. And, to ensure this, the categorical concepts under which the relevant mathematical facets of reality fall must have stipulated/legislated rules of pure use that have neither spatio-temporal nor modal restrictions. Yet, when expressed in the object-language, these stipulations amount to the relevant mathematical facets of reality, *i.e.*, the relevant domains and their components, being abstract, atemporal, and amodal existents, even though, for reasons that I shall elaborate momentarily, we tend to classify them as necessary rather than amodal existents.

Moreover, if the categorical concepts under which items in the first layer of mathematical reality fall are stipulated to have rules of pure use that have neither spatio-temporal nor modal restrictions, and items in the second layer serve RFs with respect to those in the first, then the categorical concepts under which items in the second layer fall must also have stipulated rules of pure use that have neither spatio-temporal nor modal restrictions. And, what is true of the second layer is true of all subsequent layers. Thus, the rules of pure use of *all* typical mathematical categorical concepts will, when expressed in the object-language, be expressed as claims that the mathematical facets of reality that fall under them are abstract, atemporal, and amodal existents. Moreover, given this, it seems likely that even mathematical facets of reality that cannot serve RFs

<sup>32</sup> This partially accounts for the well-documented relationship between mathematical reality and a peculiarly mathematical notion of possibility.

or whose functions are not primarily representational, should any exist, would be stipulated to be abstract, atemporal, and amodal existents. So, given our pragmatic interests in mathematical domains and the relationship, discussed in §1.5, between these and the temporal and modal profiles of institutional facets of reality, it is reasonable to assume that we should have adopted conceptual stipulations that make it the case that, in the actual world, at time  $t$ , a mathematical domain,  $X$ , exists as an abstract, atemporal, and amodal entity if and only if, in this world, at  $t$ , there are people<sup>33</sup> who collectively recognize the existential DPs carried by  $X$ .

The aforementioned stipulations also ensure that pure mathematical truths are atemporal and amodal, even though it is more common for us to classify them as necessary. At least two factors contribute to this classificatory practice. The first is that the distinction between necessary and amodal existent/truth is less clear than the distinction between eternal and atemporal existent/truth, which was largely clarified in various theological debates. The second is our everyday practice, discussed in §2.3, of using alethic modal terms to communicate information about the application and coapplication conditions of our concepts/terms in a manner parallel to our use of *deontic* modal terms to avoid misconstrual of the rules of games.

Before the relevance of this second factor will be clear, I need to make explicit that, according to my account, all pure mathematical truths/features of mathematical domains are, or are consequences of, our stipulations/legislations concerning the rules of pure use of mathematical categorical concepts. Thus, when applied to mathematical concepts/terms, the practice mentioned in the previous paragraph amounts to its being appropriate to assert, for any pure mathematical truth  $p$ , ‘necessarily,  $p$ ’, which suggests that such truths are necessary. Further, since for any mathematical entity  $x$  it is a pure mathematical truth that  $x$  exists, the aforementioned practice also warrants the assertion ‘necessarily,  $x$  exists’, which suggests that mathematical entities (and the properties and relations that are inherent components of the domains to which these entities belong) are necessary existents. If one were to draw these conclusions, however, one would be moving from an internal use of ‘necessarily’ to an external one. Thus, they do not follow. Indeed, as I have argued above, pure mathematical truths are amodal and mathematical domains amodal existents.

#### 3.4. *On the Objectivity of Mathematics*

Now, having shown that my institutional account of mathematics can explain its atemporality and amodality and our inclination to classify it as necessary, it only remains to show that this account can accommodate

<sup>33</sup> One person might suffice — see footnote 28.

mathematics' objectivity. Consider the fact that the border between the U.S. and Canada runs through Lake Erie. This is an institutional fact, yet it is also objective. John Searle [1995] labels the type of objectivity possessed by this fact *epistemic* — roughly, it is a matter of objective fact that we have adopted stipulations/legislations that endow the facet of reality in question with the given features. In earlier work [2009], I argued that, typically, mathematics is epistemically objective (*e.g.*, it is a matter of objective fact that we have adopted stipulations/legislations that make it the case that  $7 + 5 = 12$  and that make it the case that 7 is a prime number). Note, however, that while it *could have been the case* that the border between the U.S. and Canada ran 100 miles north of Lake Erie, it *could not have been the case* that  $7 + 5 = 13$  or that 7 is composite. Put another way, there seem to be constraints operative on our mathematical stipulations/legislations that are not operative on at least some of the stipulations/legislations responsible for the existence of other institutional facets of reality. Intuitively, the presence of these constraints suggests that mathematics is objective in some more robust sense than the aforementioned purely epistemic one. Indeed, it is the source and nature of these constraints that Dieterle takes to be missing from my [2009] account of mathematics' objectivity.

On my current account, the difference between the two cases discussed in the previous paragraph lies in the type of function that the respective institutional facets of reality serve. The way in which we have divided pieces of land for various deontic purposes is somewhat arbitrary; that is, those very purposes could have been (roughly) equally well served by a somewhat different division of the same land. Yet, once we have fixed how a particular piece of land is to be divided for a specific deontic purpose, and so have fixed where the transition in the relevant deontology takes place, the border that marks this transition *must* be located in exactly that place; it *cannot* be anywhere else — more precisely, it cannot be anywhere else *and serve its RFs*. Likewise, the particular types of recreational and strategic functions for which we use chess could have been served by games that had somewhat different rules than the actual Laws of Chess. Yet once the Laws of Chess are fixed, so too are the collection of type-individuated games of chess; under these circumstances, that collection could *not* be any different than how it actually is *and still serve its RFs*. Moreover, similar remarks apply to single-occupant positions in organizations and abstract fictional characters.<sup>34</sup>

<sup>34</sup> This paragraph should make clear that my use of external possibility has been systematically ambiguous between different possibilities for how to perform a given function (*e.g.*, different possible ways of dividing land for some deontic purpose) and different possible functions (*e.g.*, serving RFs with respect to a larger or smaller collection of states of affairs).

These observations might *mistakenly* be taken to suggest that, while the features of institutional facets of reality that serve SFs are unconstrained, those of institutional facets of reality that serve RFs are not. Yet SFs, in combination with *extra-institutional facts*, *i.e.*, facts that obtain independently of the institution responsible for imposing the said SFs, *can* strongly constrain the features of the facets of reality that serve those SFs. To see this, consider various legal institutions. Laws serve the general function of governing social interactions to ensure that the members of the societies governed by them can gain the benefits of such interactions. We all recognize, however, that some laws perform this function extremely well, *i.e.*, do an excellent job of meeting the constraints placed on them by this function and extra-institutional facts relevant to it, while others do not. For instance, in general, laws against murder and theft are in the former category, while the laws that prohibited the consumption of alcohol in the U.S. in the early part of the twentieth century were in the latter. Moreover, laws against murder and theft are, in general, in the former category because of certain extra-institutional facts about human beings and their social situation (*e.g.*, the fact that many are envious and that many have tempers that would cause them to kill others if they did not fear the consequences of doing so). In fact, by reflecting on legal institutions, we can see that particular facets of reality serve specific SFs more or less well depending on the features of those facets of reality and extra-institutional facts relevant to those functions. Furthermore, as our discussion of mathematics will show, at least part of the reason why the features of items that serve RFs seem to be so strongly constrained by those functions and extra-institutional facts relevant to them is the relative simplicity and straightforwardness of the examples of RFs that we have considered.

Now, how does the above apply to mathematics? Well, since mathematical domains typically serve RFs, their features/institutional facts concerning them tend to be strongly constrained, *i.e.*, the features of mathematical domains/pure mathematical facts tend to be objective in a strongly robust sense. In fact, their robust objectivity tends to be diminished only by a lack of objectivity on the part of the facets of reality with respect to which they serve their RFs. Certainly this is so if the relationship between the two can be expressed by an abstraction principle, as it can for natural numbers. Thus, for instance, if arithmetic is anything but strongly robustly objective, it must be because the facts about possible finite collections with respect to which natural numbers serve their RFs are less than strongly robustly objective.

Obviously, given my observation that these facts coevolved with our understanding of the RFs of natural numbers, it is possible that they are less than strongly robustly objective. Yet even so, the features of natural numbers/arithmetic facts are still likely to be fairly robustly objective, for many such features/facts are fixed by Hume's principle and the collections

of spatio-temporal entities that we find around us. Indeed, all such features/facts are fixed by the above and an objective combinatorial notion of possibility according to which it is possible to combine any finite collection of entities with another collection that contains just one distinct entity. Put in more concrete terms, arithmetical truths, such as  $7 + 5 = 12$ , could *not* be any different from how they are and the natural numbers still serve their RFs. Similarly, the domain of complex numbers could *not* be any different than how it is and still serve as the closure of the reals under the relevant operations.

We should not be too quick to generalize from these conclusions to a conclusion about mathematics in general, however. For instance, if, as I ventured above, some pure mathematical investigations are sustained more by their aesthetic virtues than their RFs, this might diminish the robust objectivity of the products of such investigations. Moreover, it is not always the case that the relationship between a mathematical domain and the facets of reality with respect to which it serves its RFs is as tight as the one between the natural numbers and possible finite collections. Additionally, it is not always the case that the (external) possibilities with respect to which a given mathematical domain serves its RFs can be reduced to something akin to the objective combinatorial notion of possibility discussed above. Consider, for instance, set theory. Famously, naïve set theory, as grounded in the natural abstraction principle, is inconsistent. Consequently, the axiomatic set theories that have replaced this theory are not constrained solely by a naïve conception of the RFs of sets. Rather, they reflect a decision, motivated by deep mathematical considerations, to develop set theories that embed a particular mechanism for avoiding the aforementioned inconsistency. Perhaps, as Roy Cook [2011] argues, other decisions could have been made and might even have been well-motivated from alternative perspectives. Yet, if one takes the functions of sets to reflect the aforementioned mathematical considerations, which, almost certainly, set-theorists as a whole do, then, as Penelope Maddy [2011] has argued, the resulting set theories are objective. Ultimately, Maddy's argument for this conclusion consists in a defense of two theses: there are objective facts about how sets should be if they are to perform their mathematical functions optimally, and set theory accurately tracks these facts. Thus, Maddy's conclusion is that set theory is strongly robustly objective in the above sense. Moreover, by expressing her conclusion in these terms, light is shed on something that she failed to illuminate: the ultimate ground of the objectivity present in set theory, *viz.*, the important *mathematical functions* of sets.

Now, the above is not meant to commit my account of mathematics to any particular theses about the robust objectivity of any specific mathematics (*e.g.*, if Maddy's account of the objectivity of set theory is mistaken, this will not undermine my account). Rather, the above is meant to

demonstrate that my account of mathematics can accommodate whatever robust objectivity there might be in mathematics, and this, I hope, it has done.

#### 4. Conclusion

Undoubtedly, there is much more that needs to be said to fully explicate my institutional account of mathematics and much more that could be said about its treatment of the objectivity, necessity, and atemporality of mathematics; the above merely scratches the surface. Yet I hope that it suffices minimally to articulate a metaphysical perspective concerning mathematical reality and to show that that perspective is tenable at least in so far as it can, plausibly, accommodate our assessment that mathematics is objective, necessary, and atemporal.

#### REFERENCES

- BOGHOSSIAN, PAUL [1997]: 'Analyticity', in Bob Hale and Crispin Wright, eds, *A Companion to the Philosophy of Language*, pp. 331–368. Oxford: Blackwell.
- COLE, JULIAN [2008]: 'Mathematical domains: Social constructs?', in Bonnie Gold and Roger Simons, eds, *Proof and Other Dilemmas: Mathematics and Philosophy*, pp. 109–128. Washington, D.C.: Mathematical Association of America.
- [2009]: 'Creativity, freedom, and authority: A new perspective on the metaphysics of mathematics', *Australasian Journal of Philosophy* **87**, 589–608.
- COOK, ROY [2011]: 'Logicism, separation, and complement'. Presented at the Twelfth Annual Midwest Phil. Math. Workshop, University of Notre Dame, November 5, 2011.
- DEVITT, MICHAEL, and KIM STERELNY [1999]: *Language and Reality*. 2nd ed. Cambridge, Mass.: MIT Press.
- DIETERLE, JILL [2010]: 'Social construction in the philosophy of mathematics: A critical evaluation of Julian Cole's theory', *Philosophia Mathematica* (3) **18**, 311–328.
- ELDER, CRAWFORD [2004]: *Real Natures and Familiar Objects*. Cambridge, Mass.: MIT Press.
- FIELD, HARTRY [1980]: *Science without Numbers*. Princeton, N.J.: Princeton University Press.
- LENG, MARY [2010]: *Mathematics and Reality*. New York: Oxford University Press.
- MADDY, PENELOPE [2011]: *Defending the Axioms: On the Philosophical Foundations of Set Theory*. New York: Oxford University Press.
- REA, MICHAEL [2002]: *World without Design: The Ontological Consequences of Naturalism*. New York: Oxford University Press.
- SEARLE, JOHN [1995]: *The Construction of Social Reality*. New York: The Free Press.

- [2010]: *Making the Social World: The Structure of Human Civilization*. New York: Oxford University Press.
- SEARLE, JOHN, and BARRY SMITH [2003]: 'The construction of social reality: An exchange', *American Journal of Economics and Sociology* **62**, 285–309.
- SIDELLE, ALAN [1989]: *Necessity, Essence and Individuation: A Defense of Conventionalism*. Ithaca, N.Y.: Cornell University Press.
- SIDER, THEODORE [2003]: 'Reductive theories of modality' in Michael Loux and Dean Zimmerman, eds, *The Oxford Handbook of Metaphysics*, pp. 180–208. New York: Oxford University Press.
- THOMASSON, AMIE [1999]: *Fiction and Metaphysics*. New York: Cambridge University Press.
- [2003]: 'Realism and Human Kinds', *Philosophy and Phenomenological Research* **67**, 580–609.
- [2007]: *Ordinary Objects*. New York: Oxford University Press.
- [2010]: 'Modal normativism and the methods of metaphysics', *Philosophical Topics* **35**, 135–160.
- YABLO, STEPHEN [2005]: 'The myth of the seven' in Mark Kalderon, ed., *Fictionalism in Metaphysics*, pp. 88–115. New York: Oxford University Press.