Social Construction, Mathematics, and the Collective Imposition of Function onto Reality

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Abstract: Stereotypes of social construction suggest that the existence of social constructs is accidental and that such constructs have arbitrary and subjective features. In this paper, I explore a conception of social construction according to which it consists in the collective imposition of function onto reality and show that, according to this conception, these stereotypes are incorrect. In particular, I argue that the collective imposition of function onto reality is typically non-accidental and that the products of such imposition frequently have non-arbitrary and objective features. These conclusions are interesting in and of themselves since they debunk important aspects of our socially constructed conception of social construction. Yet, additionally, they have important implications for the viability of mathematical social constructivism since resistance to such constructivism is frequently grounded in the observation that mathematics is non-accidental, non-arbitrary, and objective. As a secondary focus, I explore these implications in this paper.

Keywords: Social Construction • Social Reality • Mathematics • Function • Objectivity

It is a contingent truth that we socially construct reality. As such, in one sense, social constructs exist and have their respective features only contingently. Yet in the context of a "widespread impression that social constructionists are anti-rationalist, anti-realist and anti-objectivist" [Boghossian 2001] and the thought that social constructs are "quite bad" [Hacking 1999, p. 6], it is easy to confuse this truth with some falsehoods: that the existence of social constructs is accidental and that their features are necessarily arbitrary and subjective.[1] In this paper, I argue that these theses are false by showing that, according to my favored account of social construction, the existence of most social constructs is robust[2] rather than accidental and that many of the features of such constructs are non-arbitrary and objective.

These conclusions are interesting in and of themselves since they debunk the aforementioned aspects of the predominant socially constructed conception of social construction. Yet,

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[1] In these theses, I take both the accidental nature of the existence of social constructs and the arbitrariness of their features to be expressions of there being no rational basis for them existing or having the features that they do. As such, these theses are in line with the anti-rationalism highlighted by Boghossian.

[2] Robustness is a complicated phenomenon. Very roughly, however, a social construct is robust if it is the product of robust social construction; social construction is robust if our sustained engagement in it is non-accidental in the sense that there is a rational basis for this engagement.

additionally, they have important implications for the viability of mathematical social constructivism since resistance to such constructivism is frequently grounded in the observation that mathematics is non-accidental, non-arbitrary, and objective.[3] I am interested in defending mathematical social constructivism. So throughout, as a secondary focus, I am explicit about the implications of my arguments for such constructivism.

With this in mind, let me begin with an outline of what I take to be a defensible form of mathematical social constructivism.[4] This constructivism relates entirely to the *metaphysics* of mathematics and provides an alternative to realist and nominalist accounts of this metaphysics. This is not to say that there are no other features of mathematics that are socially constructed; almost certainly there are (e.g., key concepts and definitions). Yet this constructivism's focus is on the construction of mathematical *reality*.

According to this constructivism, mathematical reality is part of *social reality*—the portion of reality that depends for its existence on collective intentionality. Moreover, as John Searle [1995, 2010] argues, the social construction of social reality consists in the collective imposition of function onto reality. Thus, the central thesis of this constructivism is: mathematical reality is the product of our collectively imposing function onto reality, where the functions in question are surrogacy functions with respect to (logically) possible systems of objects.[5]

According to this constructivism, the social construction of mathematical reality has its origins in operations that we perform in everyday life (e.g., counting). It proceeds from these origins by an iterative process that involves generalization and extension. In some cases, these generalizations and extensions are motivated by potential applications inside or outside of mathematics, but in others, they are recognized to be possible and explored simply out of intellectual curiosity. In virtue of differing assessments of what types of generalizations and extensions are possible, classical and constructive mathematicians differ in their assessments of what systems of objects are logically possible. Typically, however, both groups socially construct a mathematical reality to serve surrogacy functions with respect to the systems of objects that they take to be logically possible.[6]

The constructivism under discussion is neutral concerning the dispute between constructive and classical mathematicians. Yet, throughout, I assume that the majority of classical mathematical statements have objective truth values. I adopt this assumption to demonstrate that even those who take the majority of classical mathematical statements to have objective truth values can endorse the constructivism under discussion. My assumption that the majority of classical mathematical statements have objective truth values amounts to the assumption that, in most cases, classical mathematicians make objectively correct assessments about what systems

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[3] See, for instance, [Dieterle 2010].
[4] See my [2013] for further details.
[5] See §1 for a discussion of surrogacy functions.
[6] I write of *a* mathematical reality in this sentence just as one might write of *a* legal system. When I write simply of mathematical reality, I assume that there is a unique mathematical reality that has been constructed by classical mathematicians and take my claims to concern this reality.

of objects are logically possible, where, roughly, such an assessment is objective if, independently of it, reality has some feature that makes it correct.[7]

   In understanding the constructivism under discussion, it is also helpful to note that, in contrast to mathematical reality, it maintains that, in most respects, physical reality exists quite independently of intentionality, though it does recognize that there are aspects of physical reality that depend on intentionality.[8] Further, while the everyday categories and concepts that we employ in our representations[9] of reality are socially constructed, this constructivism maintains that, for most such concepts (e.g., those relating to medium-sized natural objects and artifacts, and institutional items such as marriages and corporations), there are determinate and intersubjectively agreed upon criteria for whether they apply in a given circumstance and, where appropriate, whether they coapply in a pair of such circumstances. For instance, if we consider the concept 'electrical appliance within my view', there are now four items to which this concept applies (a television, a printer, a scanner, and a laptop); moreover, all four are such that, had I considered the application of this concept at any time in the last year while sitting where I am now, it would have applied to them then just as it does now. Those interested in the details of an account along these lines of how ordinary, everyday concepts should be treated should consult Amie Thomasson's *Ordinary Objects* [2007].

   One consequence of Thomasson's metaontological perspective is that, for most objects, whether or not they exist can be assessed relatively easily. One must simply identify the categorical concept under which the object in question falls, use one's understanding of that concept to identify its application conditions, and assess whether these conditions obtain. For ordinary, everyday concepts, the last step is usually trivial. Take, for instance, the issue of whether a corporation exists. Typically, this will be settled by whether an appropriate individual (e.g., a person, partnership, or corporation) executed and filed articles of incorporation in a particular jurisdiction (e.g., the State of California). Provided that such an individual undertook such an action and that nothing special has taken place in the interim (e.g., it being dissolved), the corporation in question will exist. Further, its existence will not be, as some passages in

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[7] To employ terminology that I explicate and discuss later on, I am assuming that there are ontologically objective facts about what systems of objects are logically possible.

[8] Accordingly, the constructivism under consideration is more moderate than that developed and defended by, for instance, Christian Hennig [2010].

[9] I use 'represent' and its derivatives in the same way that Searle [2010, Chapter 2] does, i.e., to mean that the state or act in question has conditions of satisfaction. According to Searle, intentional states and speech acts have two features—content and direction of fit—that combine to supply them with conditions of satisfaction. For basic intentional states and speech acts, the contents in question are propositional and consist of a way the world could be. Moreover, such states and acts have four directions of fit: downwards, where fit is a matter of the state or act's content matching how the world is (e.g., "I am editing this paper"), upwards, where fit is a matter of the world coming to match the state or act's content (e.g., "Read this paper"), presup, where fit is a matter of the state or act's content presupposing the world to be a certain way (e.g., "I am proud that I am the author of this paper"), and bidirectional, where fit is a matter of the state or act having upward and downward directions of fit simultaneously— see the examples of declarations later in the main text. The conditions of satisfaction of downward direction of fit states and acts are usually called truth conditions, while those of upward direction of fit states and acts are sometimes called fulfillment conditions.

Searle [2010, p. 201] suggest, a "massive fantasy", even though one of the conditions that must obtain in order for a corporation to exist is that its existence be collectively recognized or accepted. In fantasy, fiction, and other types of pretense, concepts with predefined or stipulated application (and coapplication) conditions are applied (and coapplied) in a context of pretense. Such a context is conspicuously absent when people undertake the actions responsible for institutional items such as marriages and corporations existing. With care, Thomasson's metaontological perspective can be extended to cover the concepts that appear in our scientific theories; it can also be extended to cover the kinds of surrogates highlighted in §1.[10]

With this outline in place, let us look ahead. In §1, I explore and elucidate social construction conceived of as the collective imposition of function onto reality. In §2, §3, and §4, I explicate robustness, non-arbitrariness, and objectivity and, respectively, argue for the robustness of various social constructs and defend the thesis that the features of many such constructs are non-arbitrary and objective.

1. The Collective Imposition of Function onto Reality

As I use *function*, it is roughly synonymous to teleological role—a role that promotes some end(s), goal(s), or purpose(s). A facet of reality[11] can have, serve, or perform a function[12] (in a context) if it can occupy a role that promotes the relevant end(s), goal(s), or purpose(s) (in that context), or, put another way, if it is capable of *functioning* in a way that promotes the relevant end(s), goal(s), or purpose(s) (in that context).[13] A facet of reality *has*, *serves*, or *performs* a function (in a context) if it in fact occupies a role that promotes the relevant end(s), goal(s), or purpose(s) (in that context), which it can do even if it is not, at the time, functioning in the relevant way (e.g., chairs typically serve the function of being objects on which people can sit even when no one is sitting on them).[14]

In representing a facet of reality as having, serving, or performing a function, one is importing *telos*—end(s), goal(s), or purpose(s)—into one's representation of it. Only beings with particular types of intentional states have ends, goals, and purposes inherently. Thus, ends, goals, and purposes are not inherent features of facets of reality that exist independently of intentionality. Nonetheless, one may still represent such facets as having, serving, or performing particular functions since one may import *telos* into one's representations of them (e.g., the trunks of some fallen trees serve the same or similar functions to chairs, while the hearts of living organisms serve the function of pumping blood around the organisms' bodies). Further, it is not only contemporary facets of reality that exist independently of intentionality that may be represented in functional terms; to use an example suggested by an anonymous referee, the function of mitochondria is to produce energy. Moreover, this was just as much their function

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[10] See my [2013].

[11] I use *facet of reality* to pick out items in *any* metaphysical category (e.g., objects, properties, relations, events, facts, states of affairs, processes, etc.).

[12] Throughout, I use the phrases "serve a function" and "perform a function" interchangeably.

[13] I place "in a context" and "in that context" in parentheses to indicate that relativity to context is usually implicit in our representations of functions rather than explicit.

[14] Henceforth, I drop explicit mention of context unless it is required.

before complex organisms with intentionality arose as it is today. Thus, like colors,[15] *telos* can be appropriately imported into our representations of facets of reality that existed before human beings did.[16]

The examples in the previous paragraph demonstrate that some facets of reality can serve specific functions independently of any actions. Other facets of reality, however, can only serve specific functions in virtue of agents undertaking certain actions (e.g., man-made chairs can only serve their primary functions since one or more people causally manipulated reality to bring into existence facets of it to have physical characteristics appropriate to serving these functions). When a facet of reality serves some function and no action was required in order for it to be able to serve that function, say that that function was *assigned* to it.[17] When a facet of reality serves some function and the action(s) of some, perhaps single-membered, group were required in order for it to have characteristics that made it able to serve that function, say that that function was *imposed* onto it. When the group in question contained more than one member, say that that function was *collectively imposed* onto that facet of reality.

The examples of functions that I have provided so far are all such that facets of reality can serve them in virtue of having appropriate physical characteristics; label such functions *physical*. Other examples of physical functions are readily available (e.g., artifacts such as cars, clothes, homes, DVDs, and scissors all serve physical functions). Physical functions do not exhaust the functions that we impose onto reality, however. As Searle [2010, p. 7] observes, "humans have the capacity to impose functions on objects and people where the objects and the people cannot perform the functions solely in virtue of their physical structure." Here are some examples: the functions of legal tender in the European Monetary Union, the US President, the black king in a game of chess, and limited liability corporations. These examples are instances of what Searle [1995, 2010] labels *status functions*, functions that facets of reality serve in virtue of having a collectively recognized or accepted[18] status—a *SF status*—that carries certain *deontic powers*,[19] i.e., rights, responsibilities, duties, obligations, requirements, entitlements, permissions,

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[15] I use the analogy with color since, before there were human beings, there were objects with the same kinds of surface reflectance profiles that the objects around us today have. As such, independently of human beings, these objects were apt to be described as having particular colors even though this representation of them is dependent on human physiology. Similarly, early mitochondria were causally related to their surroundings in ways that made them apt to be described as serving the function of producing energy even though this representation of them is dependent on particular end(s), goal(s), and purpose(s) that did not exist at that time.

[16] Searle [2010, Chapter 3, §VII] makes essentially all of the points made in this paragraph in connection with his claim that functions are "intentionality-relative".

[17] Strictly, the assignment of a function to a facet of reality requires that *no additional action* be needed in order for that facet to be able to serve that function (e.g., one might assign the functions of a paperweight to an artifact such as a stapler).

[18] Henceforth, I write simply "recognized" rather than "recognized or accepted". Moreover, following Searle [2010, pp. 8, 57-8], I do not take collective recognition to entail positive endorsement. As Searle [2010, p. 8] explains, collective recognition "goes all the way from enthusiastic endorsement to grudging acknowledgement …. Hatred, apathy, and even despair are consistent with the recognition of that which one hates, is apathetic toward, and despairs of changing".

[19] Following Searle [2010, p. 9], I use deontic powers to cover not only positive and negative deontic powers, but also logical permutations of such powers such as conditional or disjunctive deontic powers.

authorizations, etc. For instance, 'US President' carries a right to order the US Armed Forces into battle and a responsibility to uphold the US Constitution, while '€100 note' entitles the note's owner to exchange it for goods or services in the EMU.

Facets of reality make quite different contributions to their serving of status functions than to their serving of physical functions. Specifically, while it is a facet's physical characteristics that it contributes to its serving of physical functions, it being represented as bearing a SF status that carries deontic powers is an essential component of its contribution to the performance of its status functions (e.g., it is essential to Barack Obama performing his functions as the US President that he be represented as having this status and the associated deontic powers). As a result, facets of reality can serve their status functions only if their having of the related SF status and associated deontic powers is collectively recognized (e.g., if people were to stop recognizing that Obama has the status 'US President' and the associated deontic powers, he would no longer be able to perform the functions of the US President, which, in one sense, would make it the case that he were no longer the US President). Indeed, in one sense, some facet of reality has a particular SF status only if people collectively recognize that it does.

We also impose status functions onto reality in a different way to how we impose physical functions. Since facets of reality serve physical functions in virtue of their physical characteristics, we impose such functions onto reality by causally modifying the features of certain of its facets to make them appropriate. By contrast, since facets of reality serve status functions in virtue of us collectively representing and recognizing them as bearing SF statuses that carry deontic powers, we impose status functions onto reality using collective representational acts that have the same logical structure as the speech acts known as *declarations*. These represent reality as including some facet and, in so doing, bring that facet into existence (e.g., when one marries two people by declaring "I now pronounce you man and wife" in an appropriate context, or starts a meeting by declaring "let's get started" in an appropriate context). For instance, if our chess set is missing a black king, I can make a particular rock that king for the purpose of our game simply by declaring, "*this* will be the black king". Yet the representational acts that impose status functions need not be declarations in this narrow sense. A piece of rock can become the black king in our game of chess, for instance, simply by me placing it on the appropriate square at the beginning of the game. Both of the aforementioned acts have the desired effect because they result in us thinking and talking about, i.e., representing, the relevant rock as the black king. As Searle [2010, p. 13] explains, on many occasions, "we just linguistically treat or describe, or refer to, or talk about, or even think about an object in a way that creates a reality by representing that reality as created". Henceforth, 'declaration' shall refer to *all* such representational acts rather than merely declarations in the aforementioned narrow sense.

Additionally, certain status functions/SF statuses are imposed by *standing declarations* rather than by isolated declarations, i.e., declarations that remain in force for an extended period of time and, during this period, specify conditions the fulfillment of which will suffice for the existence of a certain facet of reality (e.g., the US Constitution contains the standing declarations relevant

to 'US President'). Standing declarations are also responsible for specifying the deontic powers carried by particular SF statuses; the deontic powers carried by 'black king', for instance, are specified in the Laws of Chess. Note, however, that while the standing declarations responsible for imposing status functions/SF statuses onto reality are often formally codified[20] in the way in which they are for 'US President' and 'black king', they need not be so codified (e.g., the deontic powers carried by 'member of our clique' are frequently embedded in standing declarations that are largely, or completely, uncodified).

Facets of reality that are the products of (standing) declarations and sustained in existence by collective recognition are commonly labeled *institutional* since institutions are typically responsible for their existence. Formally, in this sense, an *institution* is a plurality of activities governed at least in part by a system of standing declarations. Status functions are imposed onto reality by such systems. The relevant systems are those that impose SF statuses and specify the deontic powers that they carry; label declarations that contribute to the imposition of status functions onto reality *SF Declarations*. Searle [2010, p. 13] maintains that "all of [nonlinguistic] human institutional reality is created and maintained in existence by … SF Declarations." I disagree since I believe that there is at least a second class of functions that are imposed onto reality by systems of standing declarations: surrogacy functions.

As the *core* functions of mathematical facets of reality—those functions that sustain our collective recognition of them—are surrogacy functions,[21] we should spend some time exploring these. Before we do so, however, it will be helpful to consider the relationship between logical analysis and metaphysical classification. One approach to metaphysical classification has its origins in Frege's development of the predicate calculus and is grounded in the idea that metaphysical categories can be revealed by logical analysis; this approach is sometimes called *fantology*.[22] As an approach to the classification of facets of reality that exist at least substantially independently of intentionality, i.e., those items that exist either completely independently of intentionality or as a result of us causally or representationally modifying such items, fantology is inadequate and, so, inappropriate. By contrast, as an approach to the classification of facets of reality whose existence is entirely dependent on intentionality, fantology is both appropriate and adequate. The reason in both cases is that logical analysis reveals the structure of our *representations* of reality and only in special cases does this coincide with the ontological structure of the represented facets of reality. That fantology is both

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[20] See [Smith 2010, 2012] for interesting discussions of the roles that documents play in the construction of social reality.

[21] All relatively pervasive facets of social reality have core functions; they are the functions that, at a given time, account for the construction of the said facets by accounting for either their ongoing manufacture or our ongoing collective recognition of them. The core functions of a given facet of social reality might be only a subset of the functions that it serves. For instance, while numerous staplers function as paperweights, it is not their ability to serve this function that accounts for their ongoing manufacture. Also, the core functions of facets of social reality can change over time. For instance, the product that we now know as Listerine was originally a surgical antiseptic; it only later took on its role as a mouthwash.

[22] See [Smith 2005] for a critical discussion of fantology.

appropriate and adequate for the metaphysical classification of facets of reality whose existence is entirely dependent on intentionality is easy to see. Such items exist entirely in virtue of us representing them to exist. As such, they have an ontological structure that reflects the logical structure of our representations of them.

That fantology is an inadequate approach to the metaphysical classification of substantially independent facets of reality, i.e., those items that exist at least substantially independently of intentionality, is more controversial. I offer two justifications. First, the primary logical components of our representations of reality correlate with the metaphysical categories object, property (or attribute), and relation,[23] yet an ideal metaphysical classification of substantially independent facets of reality should include categories beyond these three (e.g., event and process). Second, it is common for us to treat substantially independent facets of reality as objects, properties, or relations in our representations of them, even when they are not. For instance, we sometimes treat processes as objects in reasoning about them even though they are not.[24] Thus, a substantially independent item being treated as an object, property, or relation in a representation does not guarantee that it is an object, property, or relation.

Two things should now be clear. First, we need at least two pluralities of metaphysical categories for classifying reality: one for classifying substantially independent facets of reality and one for classifying facets of reality whose existence is entirely dependent on intentionality. Second, the categories *object*, *property*, and *relation* should appear in both pluralities. Accordingly, in claiming that a facet of reality is an object, property, or relation, one could be claiming that it is an instance of either the relevant category of substantially independent facets of reality or the relevant category of facets of reality whose existence is entirely dependent on intentionality.[25]

Let us begin our exploration of surrogacy functions with two observations. First, many of our activities rely on our representations of the world (e.g., the activities of making inquiries, discovering truths, reasoning, analyzing, planning actions, etc.); label such activities *representational*. Second, we find it significantly easier to engage in various representational activities when their subject matter—what we are trying to inquire after, discover a truth concerning, reason about, analyze, etc.— is treated as an object in our representations. To see this, consider transitions in deontology of the type that exist between my backyard and my neighbor's and between different nations. Standardly, we represent particular tracts of land as objects upon which we impose certain types of deontology; it is merely a consequence of how these tracts are related that there are transitions in deontology between them. Yet when we engage in various representational activities concerning such transitions, we frequently employ a different system of representation that, in effect, treats such transitions as objects, i.e., borders.

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[23] These need not be the only logical components of our representations.

[24] Related examples will be provided below when I discuss surrogacy functions.

[25] Objects, properties, and relations in this second sense are sometimes labeled *logical*. Occasionally, this terminology is also applied to substantially independent facets of reality that are not objects, properties, or relations in the non-logical sense but that are treated as objects, properties, or relations in one of our representations of them.

Another way to put this is that, in various representational activities, borders serve as surrogate objects for deontic transitions. Next, consider the US Government. This is made up of people with various status functions acting in various ways. Yet when we engage in representational activities concerning it we frequently represent it as having three branches, which in turn contain such positions as that of President, Congress, and the Supreme Court. That is, we use objects— the branches and positions of the US Government—as surrogates to aid us with such activities. Moreover, the US Government is not the only organization that we treat in this kind of way; we put similar objects to similar uses in undertaking representational activities with respect to corporations, hospitals, trade unions, universities, schools, teams, other systems of government, etc. Similarly, standardly, when we are considering the sizes or cardinalities of pluralities we treat the pluralities as objects and take their sizes or cardinalities to be properties of these objects. Yet when we engage in representational activities with regard to the cardinalities of particular finite pluralities, we frequently use objects—the natural numbers—as surrogates to assist us in doing so.[26] As another example, observe that, standardly, when we are considering what is possible, we represent reality as an object that has the property of it being possible for it to have been some other way. Yet when we engage in representational activities with regard to particular possibilities, we frequently do so by using possible worlds as surrogate objects to help us in carrying out these activities.

The examples in the previous paragraph show that, when the subject matter of certain representational activities is not standardly treated as an object in our representations, we frequently engage in those activities using an alternative system of representation in which it is treated as an object. While these examples are few, reflection reveals that the practice of using objects as surrogates to aid us with representational activities is ubiquitous. For instance, type-individuated games of chess and abstract fictional characters are surrogates of this type—see my [2013] and Amie Thomasson [1999, 2003].

Debates about the ontological status of surrogate objects (e.g., legal borders, positions in organizations, natural numbers, possible worlds, type-individuated games of chess, and abstract fictional characters) are commonplace. In such debates, there are typically both realists—individuals who take the said objects to exist substantially independently of intentionality—and nominalists—individuals who deny that there are such objects. A less common position is the one that I endorse and that we shall henceforth assume: these objects are the products of systems of standing declarations that govern representational activities concerning facets of reality that are not, at least typically, the said objects; in other words, the existence of surrogate objects is entirely dependent on collective intentionality.

Some observations about this position are called for. First, I make no attempt to defend this position in this paper as it would be impossible to do so in such a short paper and, for our

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[26] Hartry Field [1980, §2] demonstrates how useful the natural numbers can be in reasoning about finite pluralities. Stephen Yablo [2005] argues for the more general usefulness of natural numbers, real numbers, and sets in various representational activities. A different yet related perspective on mathematics can be found in [Leng 2010]. My [2013] perspective differs from Field's, Yablo's, and Leng's in taking mathematical facets of reality to exist.

purposes, agreeing with this position is not as important as understanding it. Second, I wrote of systems of standing declarations in characterizing this position since surrogate objects must have specific properties and relations if they are to aid us with the relevant representational activities; in particular, they must have properties and relations that are reflections of some of the properties and relations of the facets of reality for which they serve as surrogates (e.g., the border between two nations must have the same location as the transition in deontology between those two nations if that border is to be of any use in performing representational activities concerning that transition).[27] Third, as, according to this position, surrogate objects, properties, and relations are tools to aid us with representational activities, they are the products of our collectively imposing function onto reality; label the functions that they serve *surrogacy functions*. Fourth, according to this position, surrogate facets of reality are sustained in existence by our collective recognition. In particular, in much the same way that corporations are real since we collectively recognize them to be so in order to use them as the bearers of deontic powers in certain commercial activities,[28] surrogate facets of reality are real because we collectively recognize them to be so in order to use them as surrogates in various representational activities. Fifth, as, according to this position, surrogate facets of reality are the products of systems of standing declarations that govern particular activities and are sustained in existence by collective recognition, they are institutional.

It is not merely natural numbers that I take to be surrogate facets of reality but *all* mathematical facets of reality. For the purposes of illustration, recall that Adolf Hurwitz [1898] proved that, up to isomorphism, there are only four normed division algebras over the reals and that these algebras have dimensions 1, 2, 4, and 8 respectively. In other words, he proved that, while numerous normed division algebras over the reals are (logically) possible, all such algebras have dimension 1, 2, 4, or 8, and all such algebras with dimension 1 are isomorphic, as are all those with dimension 2, all those with dimension 4, and all those with dimension 8. According to the type of mathematical social constructivism outlined in the introduction, we collectively recognize the complex numbers to serve surrogacy functions with respect to all (logically) possible norm division algebras over the reals that have dimension 2, while we collectively recognize the quaterions and octonions (or Cayley numbers) to serve such functions with respect to all (logically) possible such algebras that, respectively, have dimension 4 and dimension 8.

Before moving on, let me respond to an obvious objection to my institutional account of surrogates: since certain surrogates are amodal and atemporal existents (e.g., possible worlds and natural numbers), it is *prima facie* absurd to take them to be the products of standing declarations. However, according to Thomasson's metaontological perspective, the metaphysical features of all facets of reality are reflections of the application and coapplication conditions of

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[27] Providing the objects in question with the appropriate properties and relations requires taking into consideration the specific ends, goals, and purposes served by the imposition of the surrogacy functions in question in the context in question since which properties and relations are appropriate depends on these ends, goals, and purposes.
[28] See Searle [2010, Chapter 5] for an account of corporations that treats them in this way and my [2014] for clarification of how my Searle-inspired position understands corporations.

the categorical concepts under which they fall, and these conditions are, in turn, under our control and fixed to facilitate our ends, goals, and purposes. So, for instance, since it suits our representational ends, goals, and purposes to place neither temporal nor modal restrictions on the application and coapplication conditions of our natural number concepts, we do not, and our failure to do so, when expressed in metaphysical terms, amounts to it being the case that natural numbers are atemporal and amodal existents. For clarification, let me put my response another way. As institutional facets of reality, surrogates are the products of declarations. Consequently, their central features are as we represent them to be. Moreover, we represent institutional facets of reality to have the features that are most beneficial to them serving their core functions. Further, in some cases, surrogates can best serve their core functions if we represent them to be, and thus make them, atemporal and amodal existents.[29]

2. Robustness

Now that we know what the collective imposition of function onto reality consists in and, so, know how social reality is socially constructed, let us turn to the relationship between the collective imposition of function onto reality and robustness, non-arbitrariness, and objectivity, beginning with robustness. The collective imposition of function onto reality is contingent, yet this does not mean that it is *accidental*, i.e., that our undertaking of it lacks a rational basis. On the contrary, typically, at early stages, functions are imposed onto reality in response to them promoting, or being taken to promote, end(s), goal(s), or purpose(s) that socially powerful[30] individuals have and find important. When the imposition of a function onto reality (in a particular context) promotes, or is taken to promote, end(s), goal(s), or purpose(s) that socially powerful individuals (in that context) have and find important, label that function *robust* (in that context). Further, in recognition of the fact that functions are imposed onto reality by institutions and activities that frequently result in facets of social reality, also label robust (in a given context) any institution, activity, or facet of social reality that is involved in or is the product of the imposition of a robust function onto reality (in that context).

Some observations will be helpful in understanding the notion of robustness. First, robust facets of social reality might not serve the robust function(s) or promote the end(s), goal(s), or purpose(s) that they are taken to promote since there is a difference between individuals taking a facet of social reality to promote an end, goal, or purpose and it, in fact, promoting that end, goal, or purpose.

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[29] Consult [Searle 2010, particularly, Chapters 1 and 5] for more information about Searle's account of institutional reality, on which my account is based. Consult my [2013] for more information about my account of institutional reality, status functions, surrogacy functions (which in earlier work I called representational functions), the relationship between mathematical reality and surrogacy functions, and how Thomasson's metaontological perspective applies to surrogates.

[30] While social dynamics are complex, and societies/civilizations are never quite as simple as one group with one or more dominant members making the majority of decisions, I ask the reader to allow me to simplify this now to the following: social dominance/leadership is usually a key factor in realizing social agendas. Moreover, throughout, when I write of socially powerful individuals, I assume that they are dominant in the relevant respect(s), namely, in having or being able to secure the means (e.g., funding, education, charisma, socio-economic status) required for promoting the relevant end(s), goal(s), or purpose(s).

Second, robust facets of social reality are not inevitable, even in particular contexts, yet they are, in general, likely to be constructed in certain contexts. Such facets of reality are not inevitable since, even in contexts in which socially powerful individuals have an end, goal, or purpose that they find important, it is not inevitable that a facet of social reality that they take to promote that end, goal, or purpose will be constructed since those in that context might not have the abilities needed to construct such a facet of reality. At the same time, our long history of constructing facets of reality that were taken to promote specific end(s), goal(s), or purpose(s) that socially powerful individuals had and found important suggests that facets of social reality that have these characteristics are likely to be constructed (in specific contexts);[31] it is even reasonable to *expect* such facets of reality to be constructed (in such contexts).

Third, the social mechanisms responsible for socially powerful individuals taking a particular kind of facet of social reality to promote specific end(s), goal(s), or purpose(s) that they have and find important are frequently very complex and often include accidental or serendipitous elements. For instance, Viagra was originally intended to treat symptoms of heart disease such as angina and hypertension; it was only during early, relatively unsuccessful, clinical trials that its ability to treat erectile dysfunction was discovered. Similarly, the Kellogg brothers only hit upon the "flaking" method required for preparing Corn Flakes® as a result of being called out of the kitchen of their sanitarium and returning to find that wheat that they had cooked earlier had gone stale. Thus, the proposal that most of social reality is the result of our imposing robust functions onto reality does not entail that the construction of social reality does not include accidental or serendipitous elements. What is true is that the imposition of particular functions onto reality is only sustained for an extended period of time by sustained collective recognition or sustained manufacturing. Moreover, neither sustained collective recognition nor sustained manufacturing is likely to occur without there being some explanation of it occurring, and such explanations typically exclude either from happening by accident, i.e., without a rational basis.[32] Another way to put the point that I am making is this: at a micro level, the construction of social reality sometimes proceeds by accident and serendipity, but, on a macro level, little of social reality is accidental or serendipitous.

The nature of the contexts in which institutions, activities, facets of social reality, and functions are robust varies quite significantly, particularly in terms of specificity. In some cases, the existence of pretty much any human community in pretty much any environment will constitute a context in which a given institution, activity, facet of social reality, or function is robust; label such institutions, activities, facets of reality, and functions *globally robust* (e.g., plausibly, the institution of law is globally robust[33]). In other cases, a given institution, activity,

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[31] Examples illustrating this history are provided later in this section.

[32] Of course, some institutions and their products are sustained in existence without a rational basis since there is a kind of inertia that affects our social practices. For instance, despite the fact that many (if not most) individuals in certain parts of the world find their constructions of race and gender to be problematic in various ways, they seem unable to stop their communities (including themselves) from collectively recognizing particular races and genders.

[33] Yale's Avalon Project (http://avalon.law.yale.edu/) collects together several early legal codes.

facet of social reality, or function is only robust in contexts that are much more specific, i.e., contexts that are the products of various historical contingencies; label such institutions, activities, facets of social reality, and functions *locally robust* (e.g., plausibly, laws covering vehicular manslaughter are merely locally robust). Further, recognize that, in fact, the specificity of the contexts in which particular institutions, activities, facets of reality, and functions are robust varies along something like a continuum. Thus, in distinguishing between locally and globally robust institutions, activities, facets of social reality, and functions, I am imposing a binary distinction on something like a continuum.

Perhaps the best evidence for a particular function being robust is it having been independently imposed onto reality in a plurality of relevant contexts. Unfortunately, since nearly all functions that we have reason to believe are globally robust were first imposed onto reality prior to the systematic development of written languages, there is no reliable historical evidence of their independent imposition onto reality.[34] Despite this, there can be little doubt that various functions are globally robust, or close to globally robust. First, their imposition onto reality is both temporally and geographically pervasive. Moreover, in many cases, the pervasiveness of their imposition can be easily explained in ways that give us reason to believe that socially powerful individuals in the relevant contexts took their imposition to promote ends, goals, and purposes that they had and found important. Consider, for instance, the function 'shelter from the harmful aspects of nature'. Given the kind of beings that we are, as well as the nature of the climate and fauna in many areas of the world, it is not accidental that human beings have imposed this function onto reality in a pervasive manner. We have a physical constitution that is adversely effected both by being wet for an extended period in many climates and by being in direct sunlight for an extended period of time in many climates. Both of these effects can be mitigated by being somewhere where there is an appropriate barrier between us and the elements. This function can easily be assigned to caves and certain other naturally occurring facets of reality, but there are many areas that are nowhere near any such facets. Consequently, our ancestors who wished to inhabit those areas chose to construct facets of reality that could serve this function. Many parts of the world also contain fauna that are harmful to human beings in various ways. Once again, there are naturally occurring facets of reality that can mitigate the potential harms that many such fauna present. Yet we are not always near such facets. So, once again, our ancestors chose to construct facets of reality that could help mitigate these potential harms in similar ways to caves.

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[34] While some anthropologists believe that differences in ancient artifacts that serve the same or similar functions indicate that they were introduced independently by different cultures, there are two other equally good explanations: that they were diffused/transmitted by significant interaction between cultures or by migratory individuals. Both diffusion explanations involve cultures in adapting knowledge of how to construct artifacts to suit their local resources. Given these three equally good explanations of the data, there seems to be no definitive way to decide between them in many cases. Thus, there seems to be no conclusive historical evidence that functions that we have reason to believe are globally robust were independently imposed onto reality by distinct groups that had no contact with one another. Discussions of the debate surrounding these three explanations can be found in [Hatcher 1999], [Hervella et al 2012], [Shea 2006] and [Skoglund et al 2012].

Of course, in different places, at different times, different resources were available for the construction of shelters and shelters with differing features could stave off the differing harmful aspects of nature. Yet some features of shelters are extremely widespread (and, thus, likely, non-arbitrary). For instance, nearly all constructed shelters have roofs, probably due to the goal of them protecting people from various forms of precipitation and/or direct sunlight. Many constructed shelters also possess features similar to doors and windows. The former feature is prevalent because shelters frequently serve to keep things out, while at the same time they need to allow those whose shelters they are to enter and exit. The latter feature is common because the occupants of shelters frequently need to keep track of what is going on outside without risking the harms that might befall them were they to go outside.

The function of 'providing shelter from the harmful aspects of nature' is not the only physical function that human beings have imposed onto reality in a temporally and geographically pervasive manner. Artifacts such as knives, clothes, and bowls, all of which are the products of our imposing certain physical functions onto reality, are just as ubiquitous as shelters, as are many items that serve other physical functions. Further, given that most of these artifacts promote ends, goals, or purposes that are natural for human beings to have and find important, it is easy to see why socially powerful individuals in a wide variety of contexts would have those ends, goals, and purposes and find them important.

Furthermore, physical functions are not the only functions that we have imposed onto reality in a pervasive manner. The institutions of property, group leadership, marriage, and government, among others, are also ubiquitous, and each involves us in imposing status functions onto reality. Additionally, given that large groups of human beings seem to function more effectively with such institutions in place, it isn't difficult to explain why socially powerful individuals in a wide variety of contexts would have taken an interest in the imposition of the relevant functions onto reality.

Something of a dearth of evidence also surrounds the early imposition of mathematical functions onto reality, though it is widely accepted by historians of mathematics that certain of the functions served by arithmetic and geometry were imposed onto reality in a pervasive manner before the systematic development of written languages. The primary evidence for this is the existence of bones, dated as early as 30,000 BCE, that appear to record tallies and similarly recorded early geometric designs dated as early as 25,000 BCE. There is significantly better evidence that a more systematic development of arithmetic, in which numbers served surrogacy functions with respect to finite cardinalities, and geometry, in which idealized geometric figures served surrogacy functions with respect to the shapes of physical objects, arose during the growth of each early civilization. The best such evidence concerns the Egyptian and Babylonian civilizations, as a combination of climate and medium has left us with clear records of these two civilizations' mathematical achievements. A poorer combination of climate and medium has left us with less clear evidence that the same is true of the early civilizations that developed in China, the Indian subcontinent, and South/Central America. Moreover, it is easy to explain the interest of early peoples in the imposition of elementary mathematical functions onto reality: effective

resource management (e.g., accounting, taxation, the development of calendars) is significantly facilitated by, if not impossible in the absence of, basic arithmetic knowledge,[35] while the construction of various structures (e.g., simple shelters or grander items such as pyramids and systems of irrigation) is significantly facilitated by, if not impossible in the absence of, basic geometric knowledge.[36]

Fortunately, when it comes to locally robust functions we can draw on later, and so clearer, historical evidence. Starting first with mathematical functions, the history of mathematics is replete with important developments that were made independently by distinct mathematicians. My limited research generated the following list: Napier's and Burgi's independent introductions of logarithms, Descartes' and Fermat's independent developments of analytic geometry, Leibniz's and Newton's independent developments of the calculus, Wallis', Wessel's, and Argand's independent introductions of the complex plane as a geometrical representation of the complex numbers, and Lobachevsky's and Bolyai's independent developments of hyperbolic geometry. The independence of their invention, as well the pervasiveness of their use since the time of their invention, strongly suggests that the functions that these mathematical facets of reality serve were locally robust in the relevant contexts.

Moreover, mathematical functions and facets of reality are not the only ones that seem to be locally robust in particular contexts. Specifically, there are a variety of artifacts that are or were widely used, were independently invented by different people, and that promote end(s), goal(s), and purpose(s) that socially powerful individuals have or had and find or found important. Consider, for instance, Lippershey's, Janssen's, and Metius' independent inventions of the telescope, Morse's and Wheaton's independent inventions of the electromagnetic telegraph, Bell's and Gray's independent inventions of the telephone, and von Ohain's, Campini's, and Whittle's independent inventions of the jet engine. Moreover, certain institutional recreational functions seem to be locally robust (e.g., games that resemble football arose in several civilizations—the Mayans played pok-a-tok, the Inuits, asqaqtuk, the Greeks, episkyros, the Romans, harpastum, the Chinese, tsu chu, and the Japanese, kemari).

What all of this suggests is that functions and facets of social reality that are either globally or locally robust are common. They are imposed or constructed in contexts in which the socially powerful take them to promote end(s), goal(s), or purpose(s) that they have and find important, individuals in the relevant contexts have the abilities required to impose the said functions or

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[35] Here, I am claiming that there are a variety of forms of resource management that could account for the early imposition of basic arithmetical functions onto reality, not that all early peoples took an interest in all of these forms of resource management.

[36] All of the historical information in this paragraph is available in any contemporary introduction to the history of mathematics that deals with the relevant cultures—see, e.g., [Boyer and Merzbach 2011]. The University of St. Andrews' MacTutor History of Mathematics archive [http://www-history.mcs.st-and.ac.uk/] also contains many valuable resources, including an interesting chronology of mathematical developments. I favor explanations of early mathematical developments that emphasize their practical applications, though I do not claim that all mathematical developments occur in response to applications. It should be noted, however, that there are alternative explanations that ascribe early mathematical developments to those with more contemplative dispositions—see the early chapters of [Boyer and Merzbach 2011] for a discussion of these alternatives.

construct the said facets of reality, and the socially powerful take steps to ensure that these individuals do so. In light of these observations, the stereotype that social constructs are accidental existents, i.e., that their existence lacks a rational basis, is mistaken.

3. Non-Arbitrariness

Let us turn to the second notion of interest in this paper: non-arbitrariness. While the features of facets of social reality are, in one sense, contingent, they need not be arbitrary. In fact, as I argue below, certain features of facets of social reality are strongly constrained by their core functions and aspects of the context(s) in which they are constructed, while the core functions and context(s) in which facets of social reality are constructed provide little or no constraint over others of their features. Label features of the former type *non-arbitrary* and those of the latter type *arbitrary*. Further, note that, as I argue below, in reality, particular features of facets of social reality are more or less constrained and so more or less arbitrary, and different features of particular facets of social reality are subject to different levels of constraint and so are also more or less arbitrary.[37]

Let me now justify the contention that the imposition of various types of function onto reality is subject to varying degrees of constraint or, put another way, that the features of facets of social reality that serve particular functions are subject to varying degrees of arbitrariness. Consider first the imposition of a type of function onto reality that is subject to little in the way of constraint: the partitioning of large, relatively undifferentiated tracts of land into smaller plots in the formation of modern subdivisions. Given the lack of inherent divisions in such tracts, the relevant owners or contractors are free to partition them as they see fit. Usually, they will try to ensure that each plot can be accessed in a relatively easy way from outside of the subdivision and that the distribution of the sizes of plots within such subdivisions roughly correlates with the distribution of the sizes of plots that they believe their potential buyers will be interested in purchasing. Yet these weak constraints typically leave the relevant individuals with many possible partitions to choose between. So their actual stipulations concerning the placements and sizes of such plots are largely arbitrary. Similar remarks could be made about the partitioning of continents into nations, though it is worth noting that many borders between nations actually follow natural divisions in the land, such as rivers, lakes, mountain ranges, etc., and that it can be disastrous if insufficient attention is paid to the ethnic groupings of people when national borders are decided upon; consider Africa, for instance. Both observations suggest that the partitioning of continents into nations is not, or should not be, entirely arbitrary; certain such partitions result in nations that better serve their core functions than do others.

Next, since artifacts serve their core functions by virtue of having appropriate physical characteristics, they all have some non-arbitrary features—those relevant to serving these functions. Yet the level of arbitrariness of the features of artifacts varies. Consider paperweights,

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[37] While I do not emphasize this in the main text, it is also the case that features that are non-arbitrary at one time can be arbitrary at another. For instance, twenty years ago, it was non-arbitrary that cars should be turned on and off and locked and unlocked with a key. Yet, while many new cars still have these features, recent developments in technology make their having of these features significantly more arbitrary now than it was then.

for instance. These can be constructed out of a wide variety of materials with different shapes, sizes, colors, textures, etc. About all that is required in order for them to be able to serve their core functions is that they have a weight, size, and shape that prevents them from being easily moved by a certain level of air flow and allows them to fit on standardly sized pieces of paper. Similar remarks apply to doorstops. Thus, most of the features of paperweights and doorstops are arbitrary.

The features of other artifacts are more strongly constrained/less arbitrary than those of paperweights and doorstops, however. Knives, for instance, must be constructed out of materials that make it possible for them to have a sharp edge, while electrical wires must be made out of conductive materials. Moreover, there are artifacts with specific features that are very strongly constrained/non-arbitrary. The shapes of both keys, particularly those for relatively sophisticated locks, and arrows, for instance, are so constrained.

Furthermore, certain features of institutional facets of reality are also very strongly constrained. While the partitioning of subdivisions into plots of land and continents into nations might be largely arbitrary, once these partitions have been made, the locations of the *borders* between such plots and nations must coincide with where the relevant changes in deontology occur, for otherwise such borders would be unable to perform their core surrogacy functions. Similarly, while the deontic powers relevant to the operation of a complex organization might be distributed between individuals and pluralities of individuals in a variety of ways and this distribution might be subject to change, if a particular such distribution is in place, that distribution fixes what positions there are in the operational structure of the organization in question. Likewise, while the particular strategic, competitive, and recreational functions served by chess might be served by games governed by rules that differ somewhat from the actual Laws of Chess, those Laws precisely determine what type-individuated games of chess there are, for, if such games are to serve their core surrogacy functions, they must be correlated one-to-one and onto with the possible sequences of moves permitted by those Laws.

As one might expect in light of the examples involving surrogacy functions mentioned in the previous paragraph, certain of the features of mathematical facets of reality are also strongly constrained/non-arbitrary. For instance, since possible finite cardinalities are ordered in an ω-sequence, so too must be the natural numbers, for otherwise they would be unable to serve their core surrogacy functions. Similarly, if the sets are to serve their core surrogacy functions, they must be correlated one-to-one and onto with the possible collections for which they are surrogates.

One must be careful not to overestimate the non-arbitrariness of the features of mathematical facets of reality, however. The natural numbers are not the only mathematical objects capable of serving the core surrogacy functions of such numbers. As Paul Benacerraf [1965] famously observed, any collection of sets ordered in an ω-sequence can also serve these functions. Thus, whether or not the facets of reality that serve the aforementioned functions are set-theoretic members of one another is not determined by these functions and so is, in a sense, arbitrary. A useful way to think about this is by analogy to my earlier discussion of keys for relatively

sophisticated locks. At least for locks that aren't too sophisticated, the only major constraint that is placed on a key by its core functions relates to its shape; so long as it has an appropriate shape, it can be made out of a variety of relatively rigid materials. Similarly, the only major constraint that is operative over facets of reality that are to serve the core surrogacy functions of the natural numbers is that they form an ω-sequence. Thus, the non-arbitrariness of the fact that the natural numbers form an ω-sequence is similar in kind to the non-arbitrariness of the shape of a key for a not-too-sophisticated lock.[38]

Next, note that different facets of social reality can better or worse serve particular functions. For instance, while a butter knife might be able to serve the core functions of a flat head screwdriver, it will not do so as well as an actual flat head screwdriver of the appropriate size. Institutional facets of reality can also better or worse serve particular functions. This fact was implicit in my discussion of the division of land for various deontic purposes. As another example, consider laws. Their core functions include the general function of facilitating social interactions to ensure that various members of the societies governed by them can gain the benefits of such interactions. Yet clearly some laws serve this general function, as well as their more specific functions, better than others. Appropriately enforced laws against murder and theft, for instance, tend to serve these functions extremely well, while the laws that prohibited the consumption of alcohol in the USA at the beginning of the twentieth century served them poorly. Furthermore, in light of how well appropriately enforced laws against murder and theft serve the core functions of laws, it is, surely, non-arbitrary that nearly all legal systems include such laws.[39]

I have now highlighted two senses in which the features of facets of social reality might be non-arbitrary. First, it might be required that a given facet of social reality have a particular feature in order for it to be able to serve its core functions. Second, it might be that by having a particular feature a facet of social reality is able to serve its core functions better than it would be able to if it did not have that feature. In light of these two senses, it is clear that there is no conflict between mathematical social constructivism of the type outlined in the introduction and the fact that certain of the features of mathematical facets of reality are non-arbitrary. Indeed, since the core functions of mathematical facets of reality are surrogacy functions, we have every reason to believe that various of their features—specifically, those relevant to them being able to serve their respective surrogacy functions—are highly non-arbitrary.

4. Objectivity

Let us turn to the complicated topic of objectivity. Almost certainly, in maintaining that some statement, theory, or facet of reality is objective, different authors have ascribed different

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[38] The observations at the end of this paragraph demonstrate that the mathematical social constructivism outlined in the introduction is a variety of non-eliminative structuralism—see [Shapiro 1997] and [Resnik 1997] for defenses of realist, non-eliminative structuralisms. Exploring the relationship between this constructivism and non-eliminative structuralism goes beyond the scope of this paper.

[39] The documents in Yale's Avalon Project (http://avalon.law.yale.edu/) demonstrate that laws against murder and theft really are present in nearly all legal codes, including the earliest surviving such codes.

characteristics to the said statement, theory, or facet of reality. Our interest is in one characteristic of statements and theories that I take to be at least part of what some people have meant when they have claimed that mathematics is objective. As a starting point in identifying this characteristic, consider this passage from Searle [2010, p. 18]:

> There are at least two different senses of the objective/subjective distinction: an epistemic sense and an ontological sense. The epistemic sense has to do with knowledge. The ontological sense has to do with existence. Pains, tickles, and itches are ontologically subjective in the sense that they exist only as experienced by human or animal subjects. In this sense they differ from mountains and volcanoes, which are ontologically objective, in the sense that their existence does not depend on anybody's subjective experiences. But in addition to that, there is an epistemic sense to the distinction. Some propositions can be known to be true independently of anybody's feelings or attitudes. For example, the statement that Vincent van Gogh died in France is epistemically objective, because its truth or falsity can be ascertained independently of the attitudes and opinions of observers. But the statement "Van Gogh was a better painter than Manet" is, as they say, a matter of subjective opinion. It is epistemically subjective. … Ontological objectivity and subjectivity have to do with the mode of existence of *entities*. Epistemic objectivity and subjectivity have to do with the epistemic status of *claims*.

Searle is onto something in this passage, but there is a flaw in his discussion. In particular, he has run together two things in his characterization of epistemic objectivity: the objectivity or subjectivity of our methods for ascertaining the truth or falsity of a given content and the objectivity or subjectivity of the said content, i.e., whether or not the truth value of that content depends on the feelings, attitudes, opinions, etc. of those who are assessing it. Searle's interest is in the objectivity or subjectivity of the said content. Some critiques of science, however, have focused on the objectivity of the methods used by scientists in ascertaining the truth values of certain contents.[40] So, Searle's two senses of the objective/subjective distinction should be replaced by three: ontological, semantic, and epistemic.

Only the ontological and semantic senses of the objective/subjective distinction are relevant to our purposes. In the terminology of this paper, it is facets of reality that are appropriately classified as *ontologically objective* or *subjective*; roughly, they are objective if they exist independently of intentionality and subjective if they do not (e.g., social reality is ontologically subjective, while items such as electrons and trees are ontologically objective). By contrast, it is propositional contents, and items that are intimately connected with such contents (e.g., sentences, beliefs, statements, and truths), that are appropriately classified as semantically objective or subjective. A content is *semantically objective* if it has its truth value independently of the judgments, preferences, attitudes, etc. of those who are assessing it, while it is *semantically subjective* if its truth value is dependent on such judgments, preferences, attitudes, etc. (e.g., the content 'human beings are reptiles' is semantically objective since it is false independently of any individual's judgments, preferences, attitudes, etc., while the content

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[40] See, e.g., [Harding 1987].

'chocolate ice cream tastes better than vanilla ice cream' is semantically subjective since its truth value depends on the preferences of the individual who is assessing it).

A clear understanding of the distinction between semantically objective and subjective contents is important for understanding the notion of objectivity of interest to us. Yet it is only a first step, for we are interested in a more refined notion. Semantic objectivity is of interest to us because it allows us to account for the objectivity of various contents that directly represent ontologically *subjective* facets of reality (e.g., 'the border between the United States of America and Canada runs through Lake Erie' and 'the United State Supreme Court, when full, has nine justices'). Yet if such contents are semantically objective, so too must be the constitutive contents of our concepts of race and gender (e.g., 'women are more nurturing than men').[41] However, classifying such contents as objective is contrary to an important, if not the most important, goal of social constructivists concerning race and gender: showing that the constitutive contents of our concepts of race and gender are not objective.

How, then, might we make semantic objectivity compatible with this goal of social constructivists concerning race and gender? We can do so by recognizing that what such constructivists are claiming in maintaining that the constitutive contents of race and gender are not objective is that their truth values are not constrained to be what they are by the features of ontologically objective facets of reality. By contrast, the truth values of other semantically objective contents (e.g., 'the Earth travels around the sun') are constrained to be what they are by the features of ontologically objective facets of reality. It is contents that are semantically objective in a constrained way that are of interest to us in this paper.

One way the features of ontologically objective facets of reality can constrain the truth values of semantically objective contents is for such contents to directly represent the said features. Yet this is not the only way. Consider, for instance, 'there is a possible world in which the Earth's orbit is roughly one kilometer further away from the Sun than it actually is'. Given the surrogacy functions of possible worlds, this content *must* have the same truth value as 'it is possible for the Earth's orbit to be roughly one kilometer further from the Sun than it actually is'. The latter content directly represents a feature that reality has independently of intentionality, so its truth value is constrained by features of ontologically objective facets of reality. Moreover, since the former content must have the same truth value as the latter, its truth value is also constrained by features of ontologically objective facets of reality.

Furthermore, the reasoning in the previous paragraph can be applied to any semantically objective content that directly represents a feature of a surrogate for ontologically objective facets of reality. The features of surrogates are direct reflections of certain of the features of the facets of reality for which they are surrogates. So, when the relevant non-surrogate facets of reality are ontologically objective, the features of their surrogates are entirely determined by

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[41] This example is taken from Jill Dieterle [2010, p. 326], who alerted me to the need for the more refined notion of objectivity that I am about to introduce by making the observation that I am about to make.

features of ontologically objective facets of reality. As such, so are the truth values of contents that directly represent the features of such surrogates.

It should now be clear that ontologically objective facets of reality do not constrain the truth values of contents by themselves, but in combination with certain functions. To understand this phenomenon more generally, observe that, for contents that directly represent ontologically objective facets of reality, two factors contribute to the latter constraining the truth values of the former: the features of the relevant ontologically objective facets of reality *and* that the core function of such contents is to directly represent those particular features. The core function of contents that are consequences of us imposing functions onto reality is to directly represent features of facets of social reality whose core functions are the imposed functions. As such, the truth values of such contents are constrained in exactly the same manner and to exactly the same extent as the relevant features of the relevant facets of social reality. Consequently, the varying degrees of constraint over the features of facets of social reality that serve particular functions highlighted in §3 correspond to varying degrees of constraint over the truth values of contents that directly represent those features of facets of social reality. When a semantically objective content directly represents a feature of a facet of social reality that is largely or entirely unconstrained, say that that content has largely or entirely *unconstrained semantic objectivity* (e.g., contents that directly represent the locations of borders between nations have largely unconstrained semantic objectivity since the division of continents into nations is largely unconstrained). When a semantically objective content directly represents a feature of a facet of social reality that is largely or entirely constrained, say that that content has largely or entirely *constrained semantic objectivity*. For instance, 'the Earth travels around the sun' and '$7 + 5 = 12$' have entirely constrained semantic objectivity. The 'Earth' content has such objectivity since it directly represents a feature of ontologically objective facets of reality, while the arithmetic content has such objectivity since it has the same truth value as 'when disjoint and non-interacting pluralities of cardinality five and seven are combined, the result is a plurality with cardinality 12' and this content directly represents a feature of ontologically objective facets of reality. Further, the system of classification just provided highlights that the type of objectivity possessed by '$7 + 5 = 12$' is more like that possessed by 'the Earth travels around the sun' than that possessed by 'the border between Canada and the USA runs through Lake Erie' and 'women are more nurturing than men'. Informally, the truth value of the first two contents is entirely constrained, while that of the second two contents is hardly constrained at all. This, of course, does not mean that the second two contents are not semantically objective. They are since the border between Canada and the USA does run through Lake Erie and, according to our gender constructions, women are more nurturing than men. Yet unlike '$7 + 5 = 12$' and 'the Earth travels around the Sun', the semantic objectivity of the second two contents is largely unconstrained.

In conclusion, let me be explicit about the implications of this discussion of objectivity for mathematics. Mathematical facets of reality are surrogates for logically possible systems of objects. Consequently, mathematical contents and the corresponding contents relating to what

systems of objects are logically possible are not merely both semantically objective, but possess semantic objectivity that is equally constrained by the features of ontologically objective facets of reality. Throughout, I have been assuming that both possess entirely constrained semantic objectivity since there are ontologically objective facts about the relevant logical possibilities. Yet, suppose that this is incorrect and, for instance, that the types of generalization and extension that are involved in the construction of mathematical reality weaken the extent to which the features of its facets are constrained. Still, this will not detract from the ability of the mathematical social constructivism outlined in the introduction to account for whatever constrained semantic objectivity mathematical contents in fact have. This is so since any weakening of the constraints that effect mathematical contents also effect the corresponding contents relating to what systems of objects are logically possible. There are really only three possibilities. First, it could be that the relevant logical possibilities are as they are independently of us. If so, all mathematical contents have entirely constrained semantic objectivity. Second, it could be that the relevant logical possibilities are constrained by ontologically objective aspects of reality but not entirely constrained; we are partially responsible for constructing them as we see fit. If so, mathematical contents have constrained semantic objectivity of potentially varying levels. Third, it could be that the relevant logical possibilities are largely or entirely constructed by us in a largely or entirely unconstrained way. If so, mathematical contents have largely or entirely unconstrained semantic objectivity. Thus, whatever constrained semantic objectivity you take various mathematical contents to have, your view is compatible with the type of mathematical social constructivism outlined in the introduction.

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